Radiative strength functions from the energy-localized Brink-Axel hypothesis

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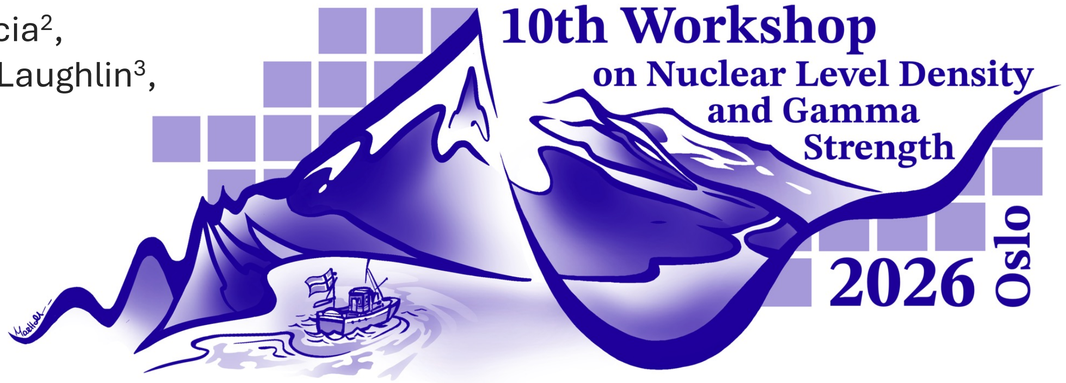
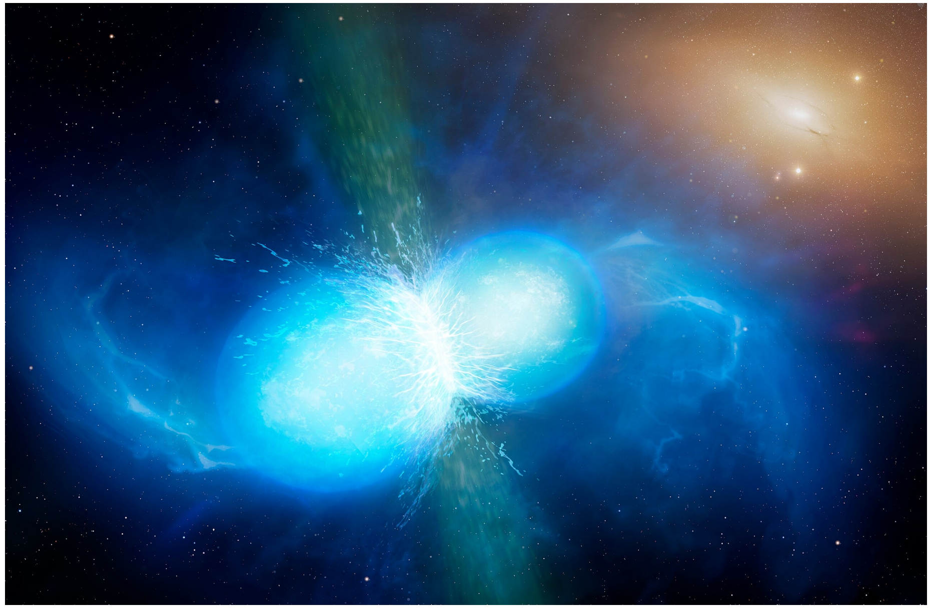


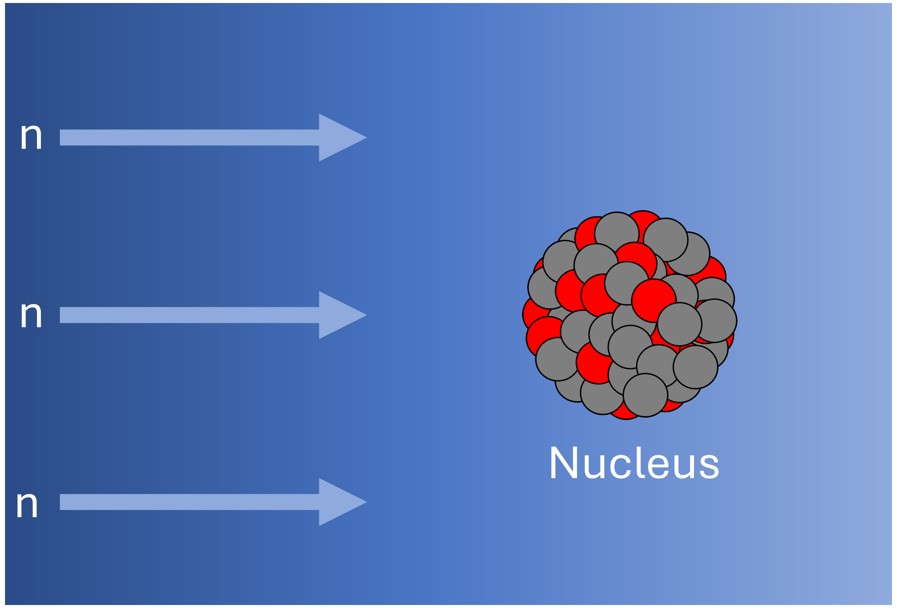
Prepared by LLNL under Contract DE-AC52-07NA27344.

How and where are heavy elements formed?

Answer "Extreme radiation environments with ultra-high neutron fluxes"



Neutron star merger



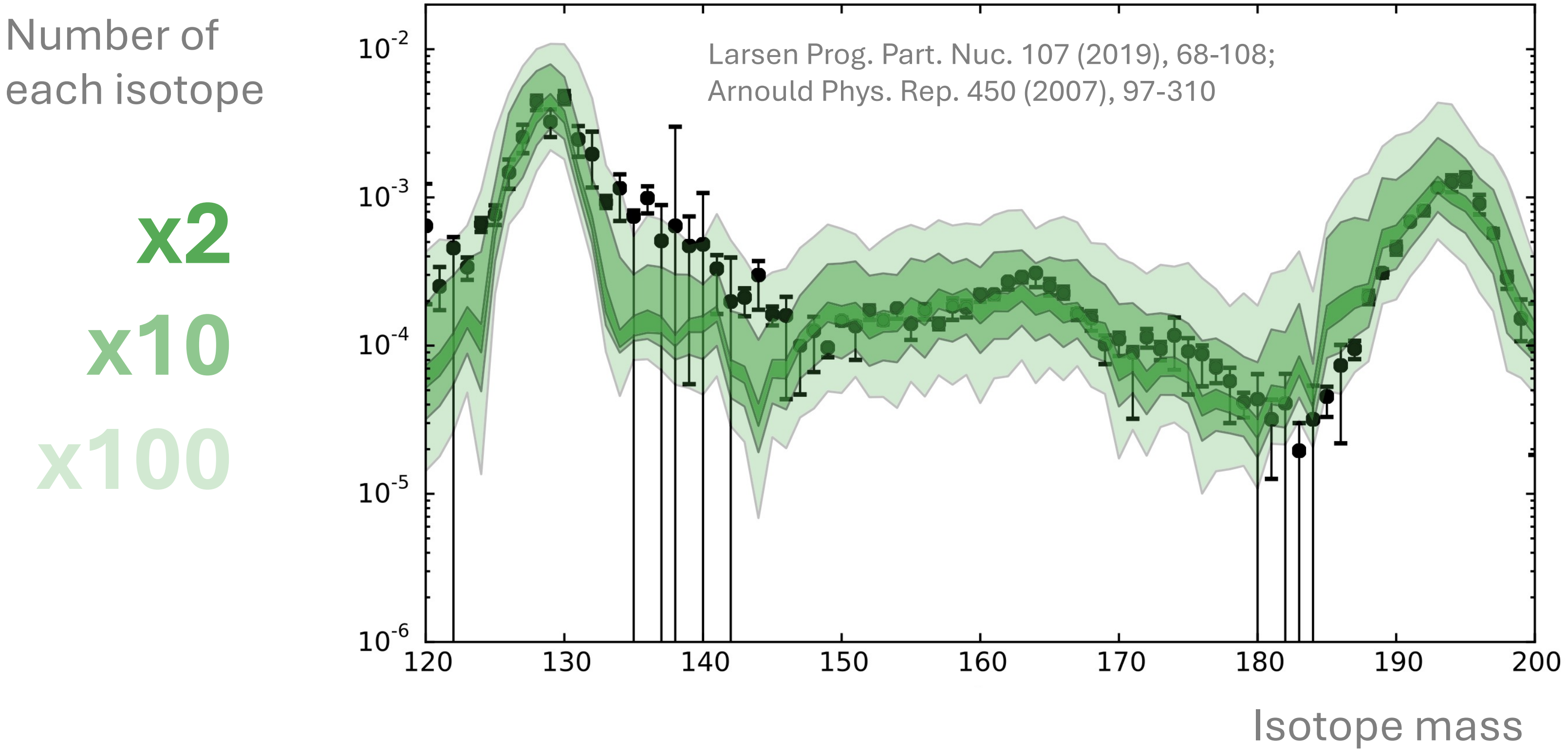
Flux > $10^{37}/cm^2/s$

(Neutron density >$10^{29}/cm^3$ @ 1GK)

Isotopic “fallout” from extreme events

The task

Given **radiochemical evidence**,
determine the **event** (neutron source) that produced it

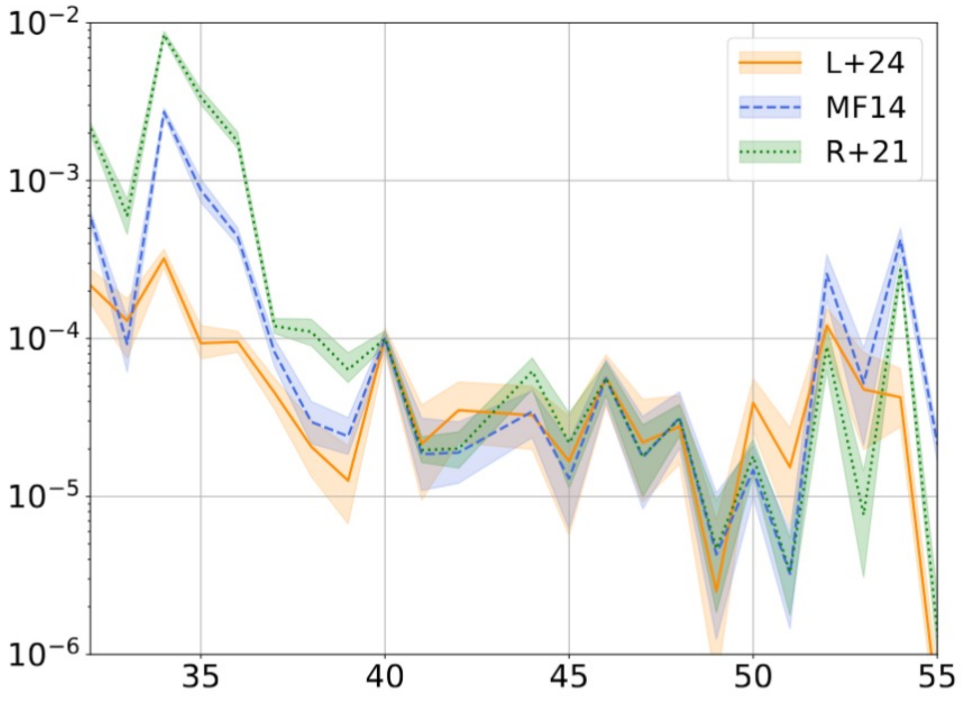


Hydrodynamics versus nuclear physics uncertainty

Task Given **radiochemical evidence**, determine the **event** that produced it

3 hydro simulations (2 NS mergers, 1 supernova) + 1 source of nuclear uncertainties*

Amount of isotope produced



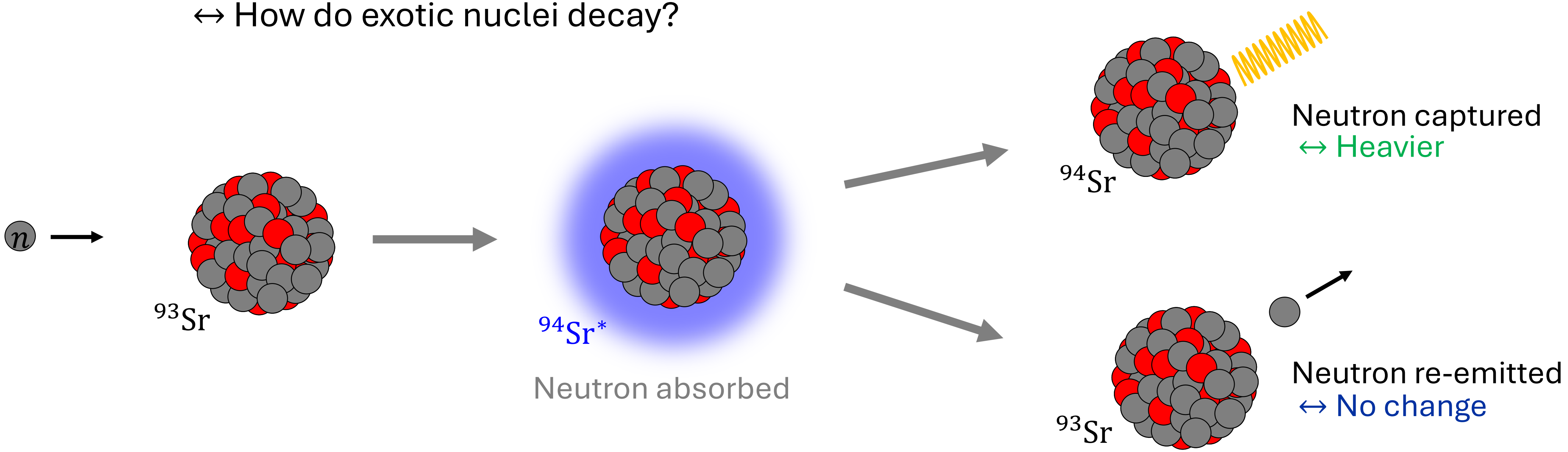
Isotope Z

Figure from Atul Kedia (NCSU) arXiv:2602.12428 (under review for PRC)

*neutron optical model

Nuclear physics needs

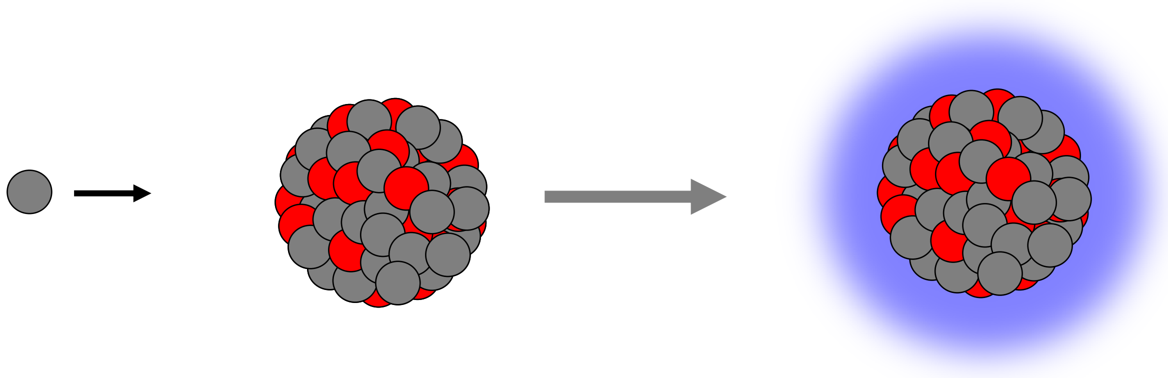
Need to know: How do exotic nuclei get heavier?
↔ How do exotic nuclei decay?



Optical model

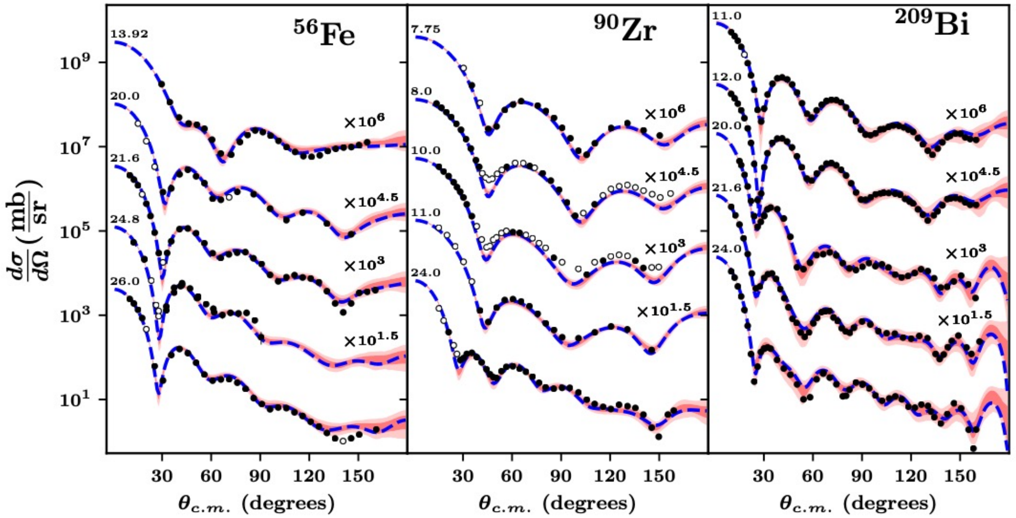
Radiative strength functions,
Level densities,
Optical model

Optical potentials with uncertainties have led the way for physics-informed UQ in reactions



Optical model

KDUQ global optical potential



From Pruitt, Escher, Rahman PRC 107 014602 (2023)

Physics-informed UQ in reactions: learn physical parameters, predict observables

$\mathrm{L}\left(\frac{d\sigma}{d\Omega}\right)$: Scattering data, $O(10^4)$

$P(\omega)$ OMP parameters ω, $O(10^2)$

$p[y'(\theta)]$: Global nucleon transmissibility, $O(10^{>3})$

Optical model

Astrophysics applications with 500+ cross sections from physics-informed uncertainties

Neutron-capture rates with UQ from the KDUQ potential*

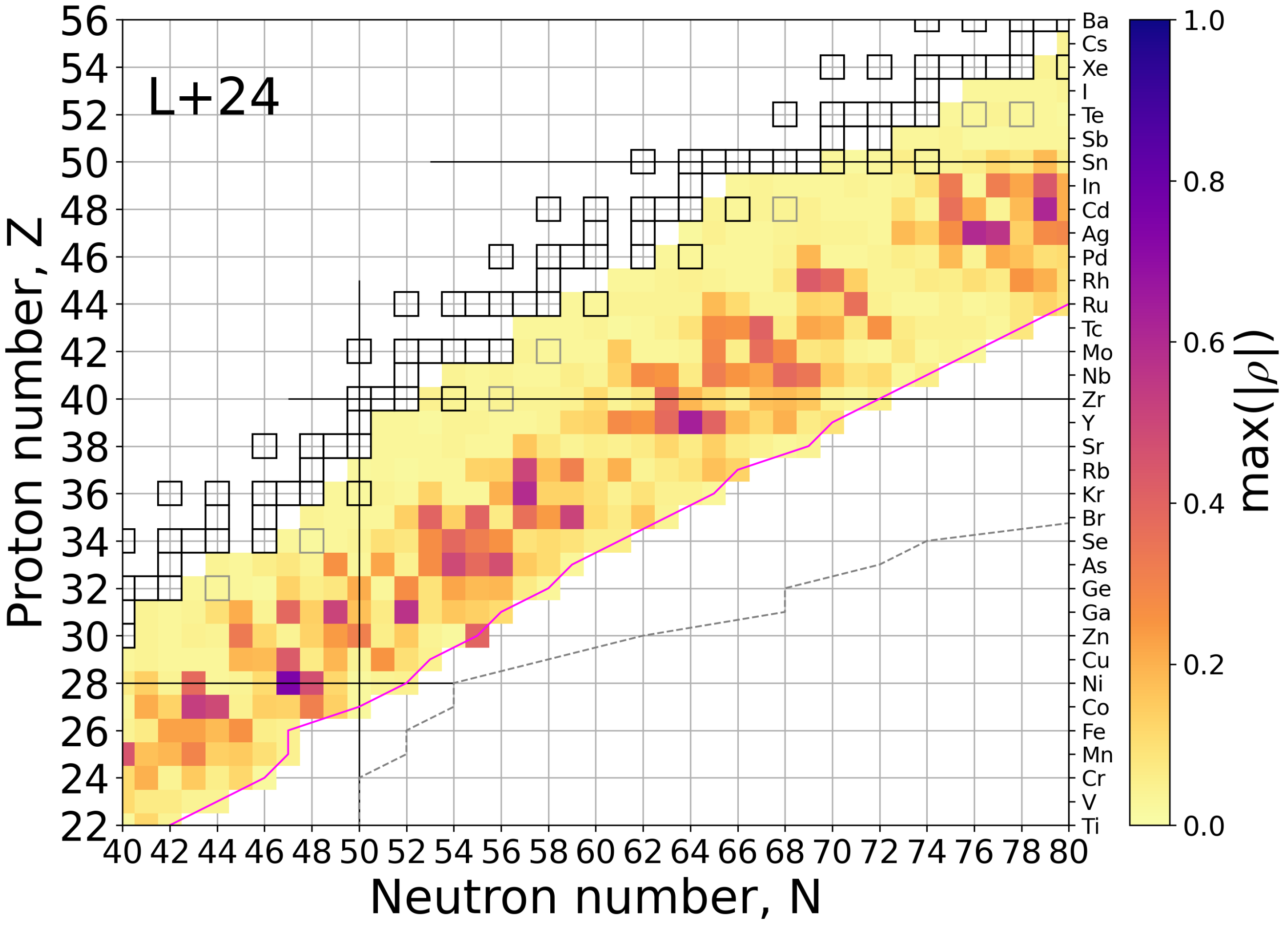


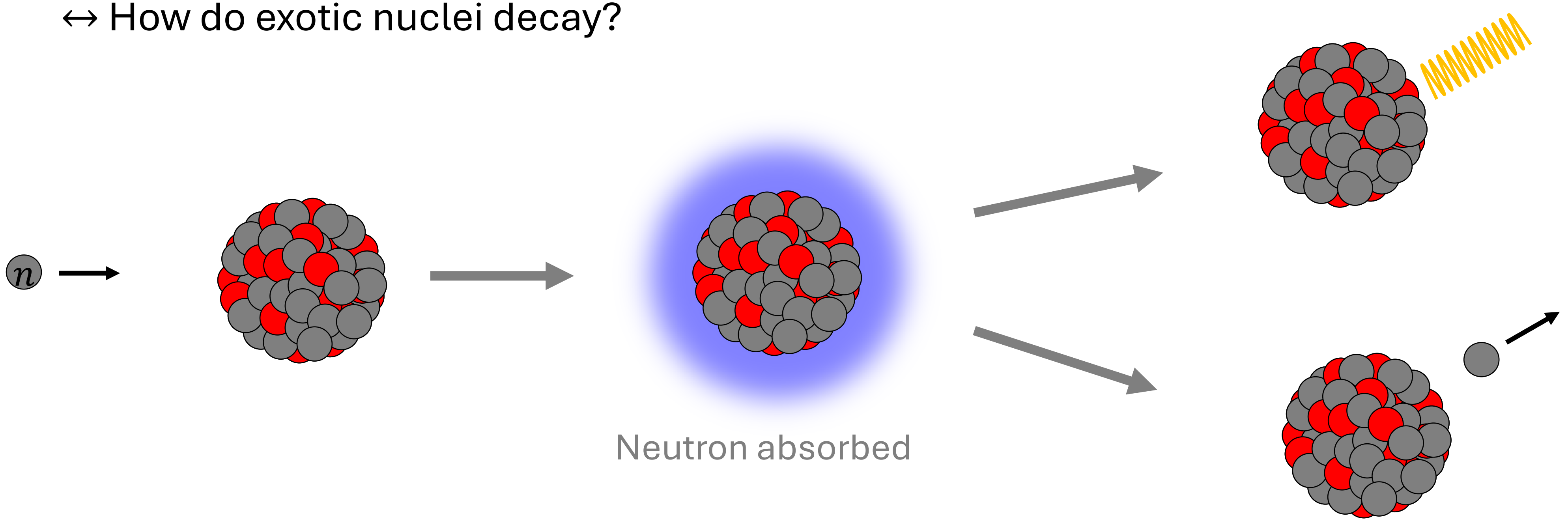
Figure from Atul Kedia (NCSU)
arXiv:2602.12428
(under review for PRC)

*neutron optical model

The other inputs for nuclear reactions

Need to know: How do exotic nuclei get heavier?

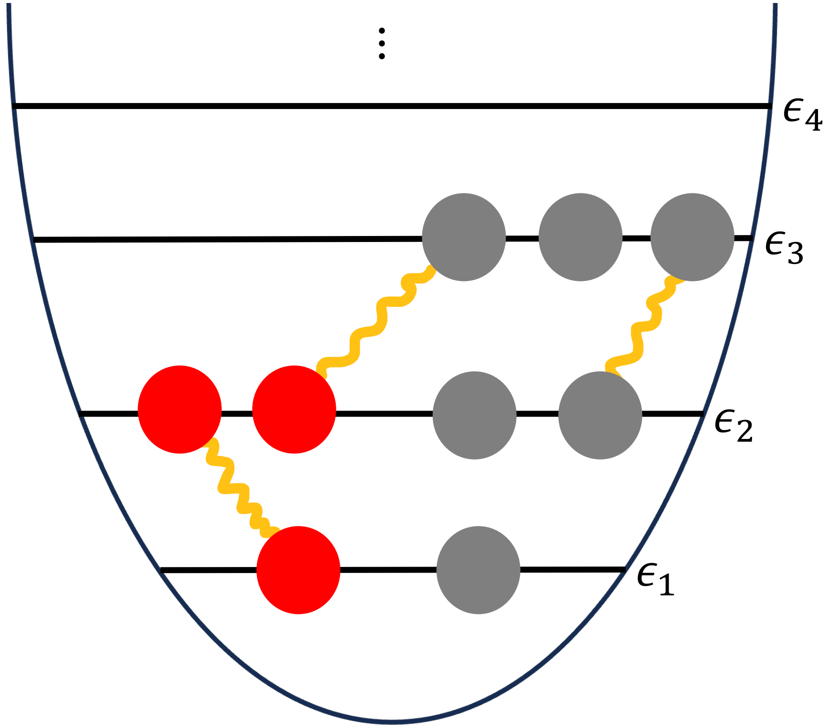
↔ How do exotic nuclei decay?

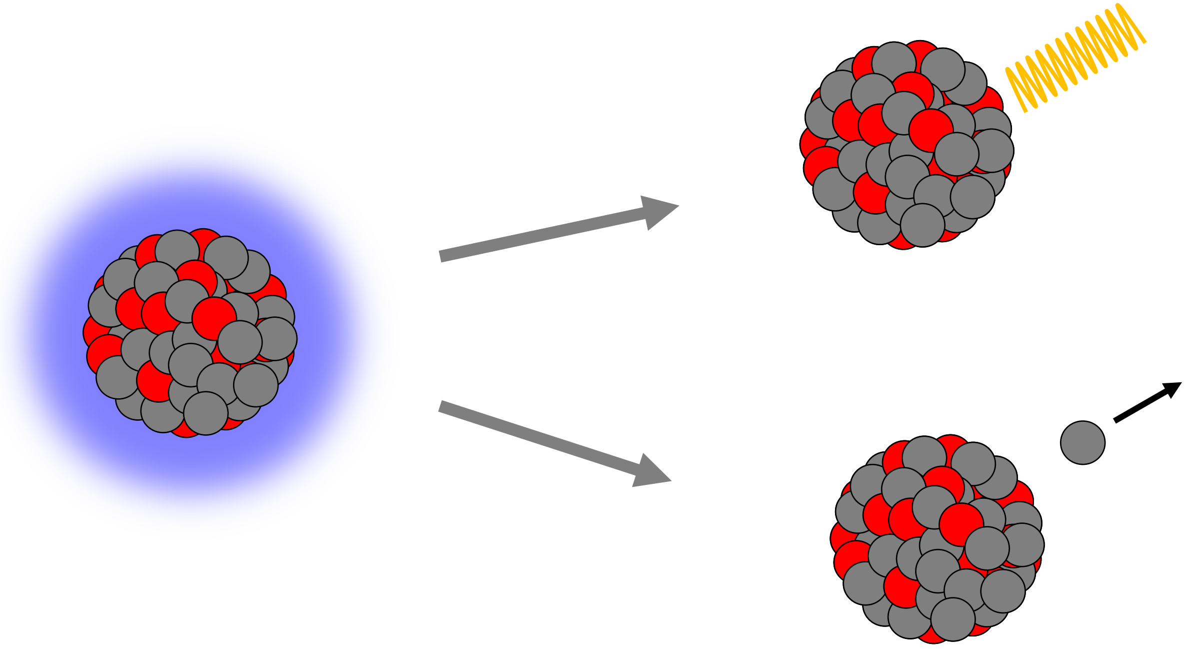


Optical model

Radiative strength functions,
Level densities,
Optical model

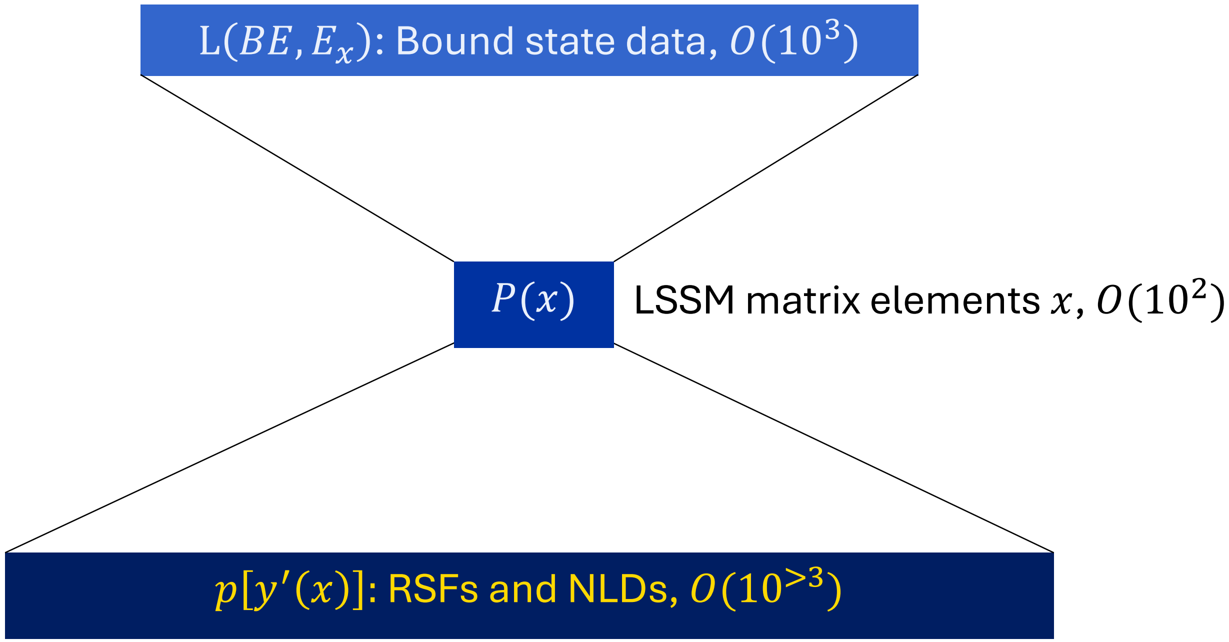
Large-scale nuclear shell model (LSSM) can provide these inputs





Radiative strength functions,
Level densities,
Optical model

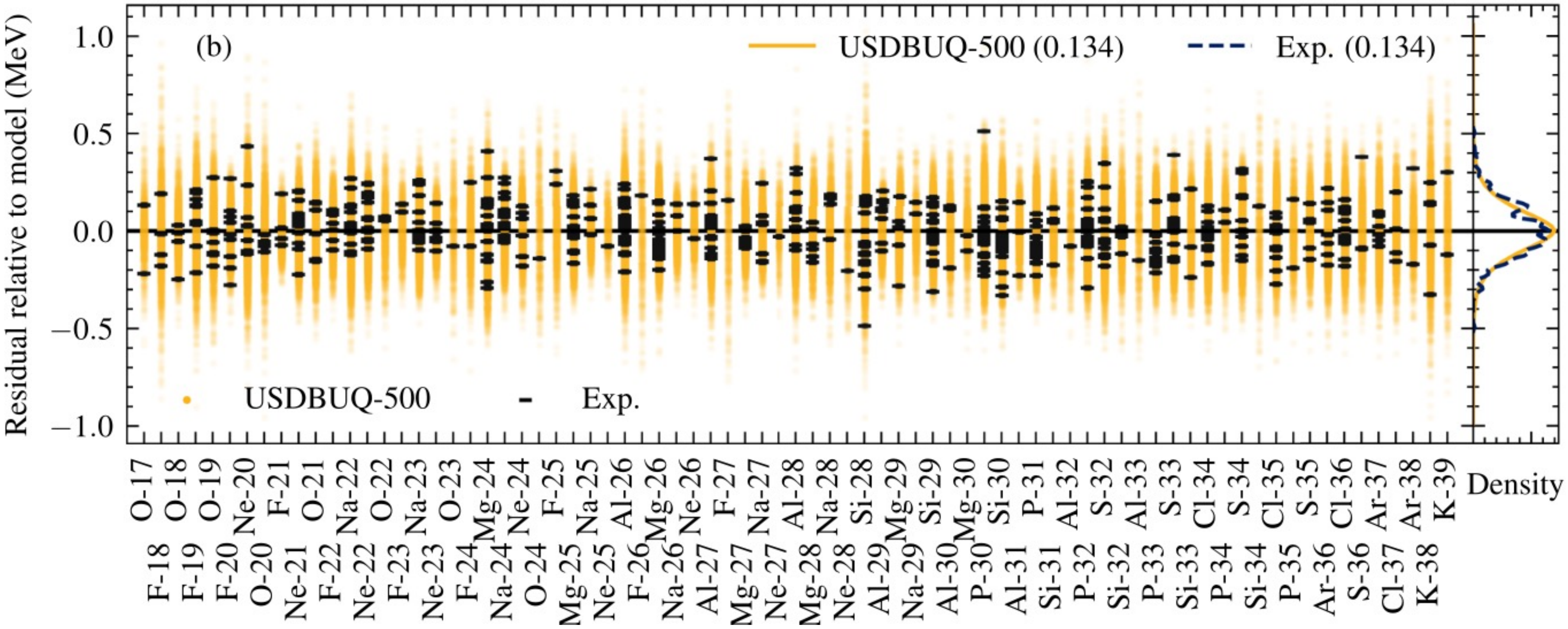
Physics-informed UQ from shell model: learn physical parameters, predict observables



Gorton, Kravvaris PRC 112, 014302 (2025)

We published the first shell model interaction with uncertainties

Distribution of energy-level predictions

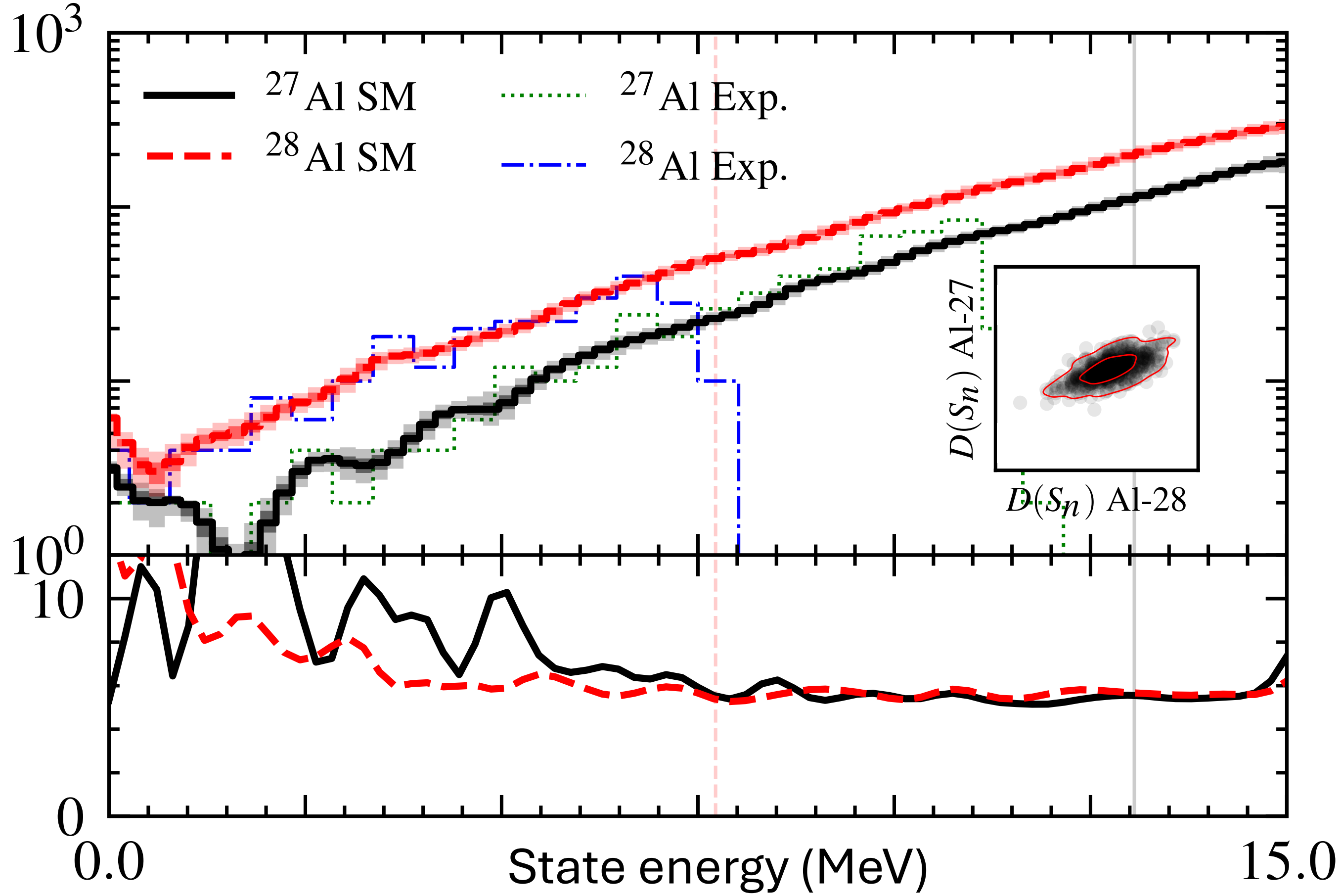


Gorton, Kravvaris PRC 112, 014302 (2025)

Now we compute structure inputs for reactions: **Nuclear Level Densities (NLDs)**

Density of states
(1/MeV)

Predicted uncertainty
(%)

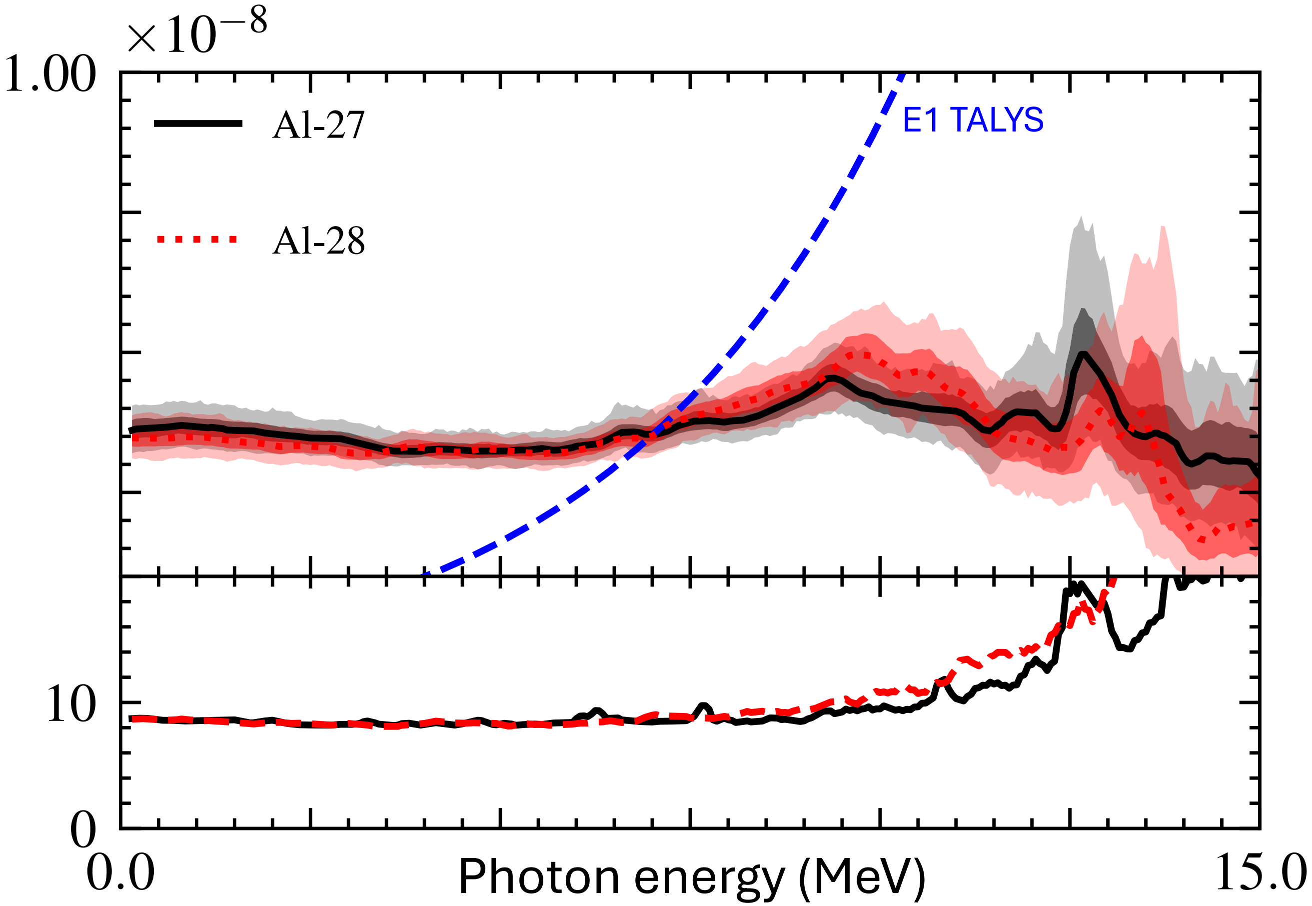


Gorton and Kravvaris (2026) arXiv:2604:09935

Now we compute structure inputs for reactions: **Radiative Strength Functions (RSFs)**

Photon strength function
(1/MeV³)

Predicted uncertainty
(%)



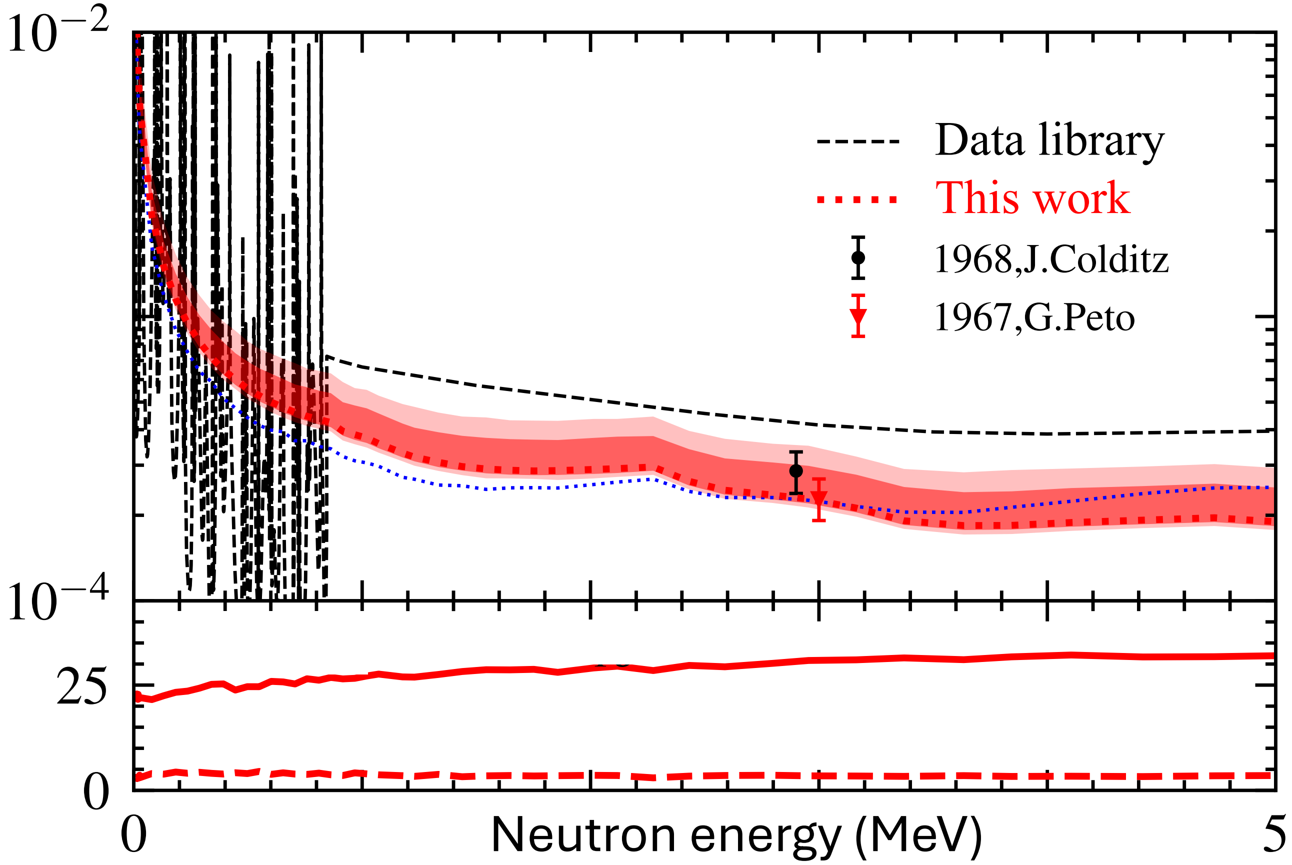
Gorton and Kravvaris (2026) arXiv:2604:09935

Developing workflows to propagate uncertainty from microscopic theory to applications

Capture cross section
(10^{-24} cm^2)

Modern
statistical methods
enhance credibility

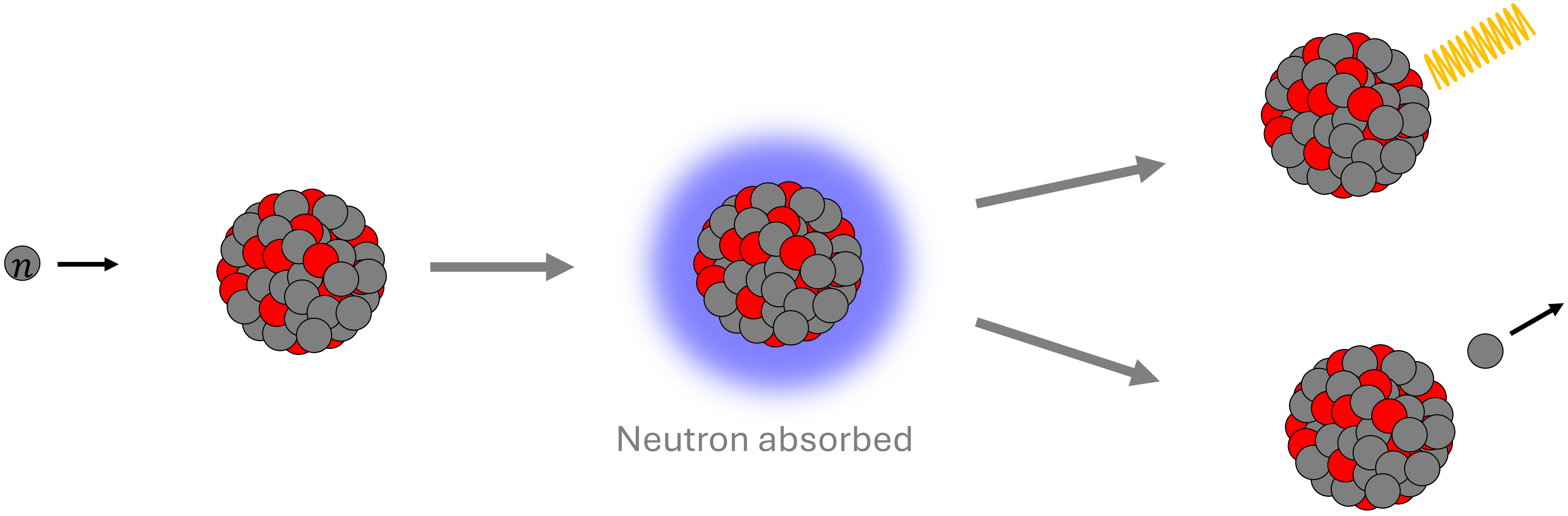
Predicted uncertainty
(%)



Gorton and Kravvaris (2026) arXiv:2604:09935


Cross section (b)
Unc. (%)
Neutron energy (MeV)
TENDL-2025
ENDF/B-VIII.1

How to compute Radiative Strength Functions with the LSSM



Optical model

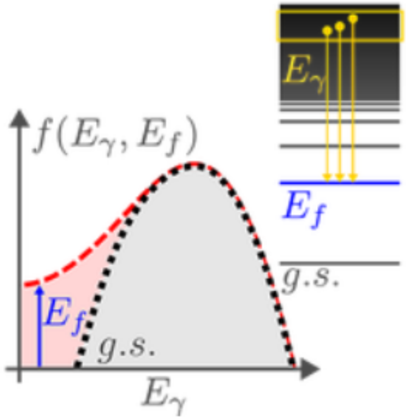
Radiative strength functions,
Level densities,
Optical model

EDITORS' SUGGESTION

Radiative strength functions from the energy-localized Brink-Axel hypothesis

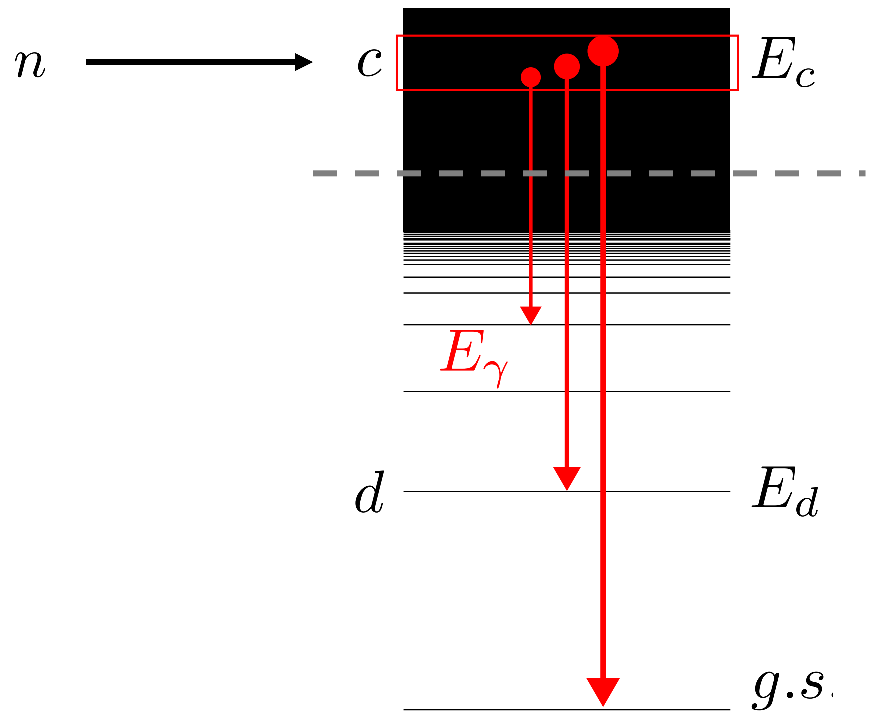
Oliver C. Gorton, Konstantinos Kravvaris, Jutta E. Escher, and Calvin W. Johnson

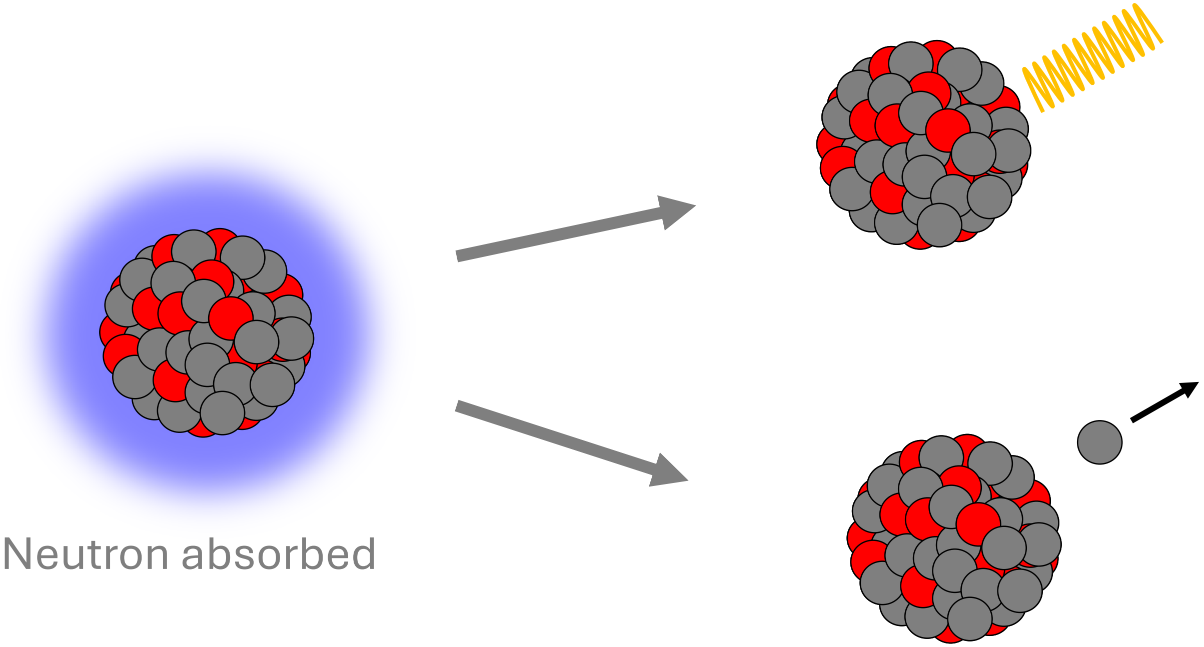
Phys. Rev. C **113**, 044327 (2026) - Published 29 April, 2026



Radiative strength functions (RSFs) are crucial inputs for statistical nuclear reaction codes but are difficult to calculate because they require wave functions of highly excited states. The authors used large-scale shell model calculations to identify a key result of the energy-localized Brink-Axel hypothesis: the shape of the RSF evolves smoothly with wave-function energy. Combined with an efficient Lanczos strength-function method, this insight leads to a practical new approach for computing RSFs which was validated for ^{24}Mg and provided novel results for ^{56}Fe. The method is expected to simplify RSF calculations while motivating the use of energy-dependent RSFs in modern reaction codes.

Radiative decay must be described for nuclei excited to 10s of MeV

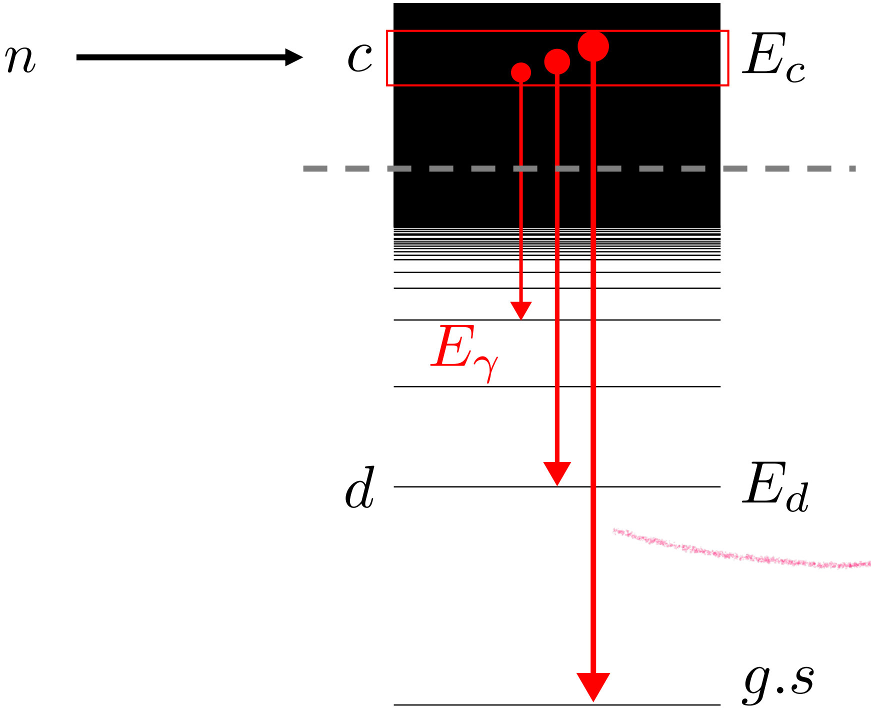




Neutron absorbed

Radiative strength functions,
Level densities,
Optical model

Many highly-excited wave functions participate

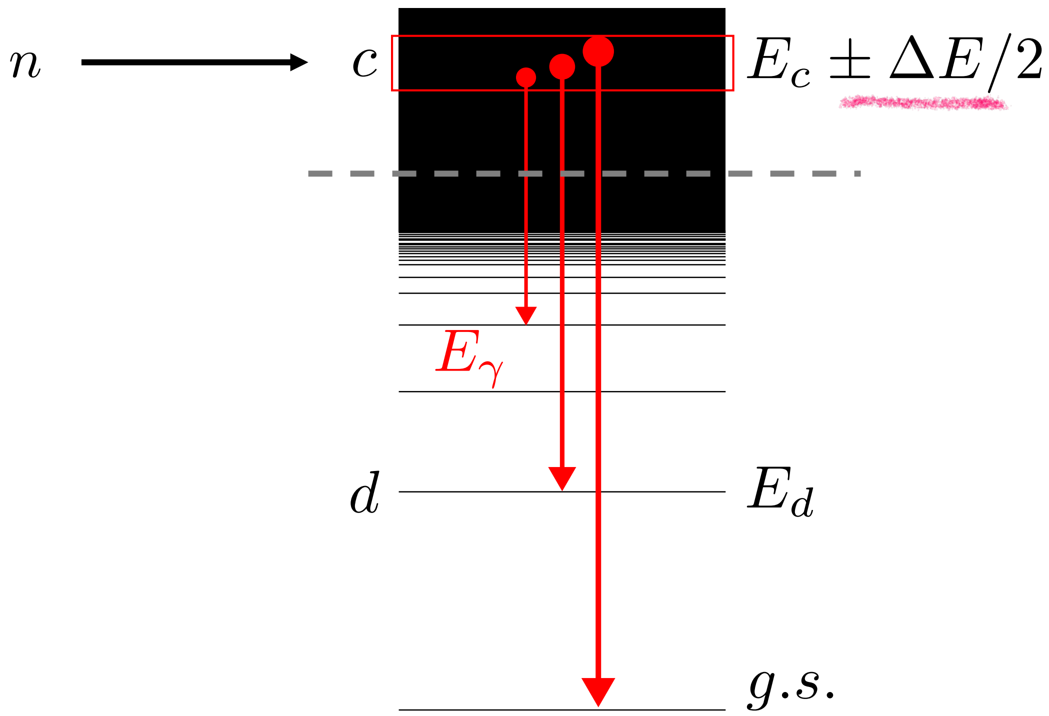


"An **exact theoretical description** of [radiative transitions to or from highly excited levels] **is impossible**, since the radiative transition probabilities can be calculated only if the nuclear wave functions are known"
– Blatt and Weisskopf, 1952

$$T_{c\to d} = \frac{2\pi}{\hbar}\left|\langle {}_d|\hat{H}_{\mathrm{ext.}}|{}_c\rangle\right|^2$$

(First-order perturbation theory)

Radiative strength functions approximate the bulk electromagnetic response of excited nuclei

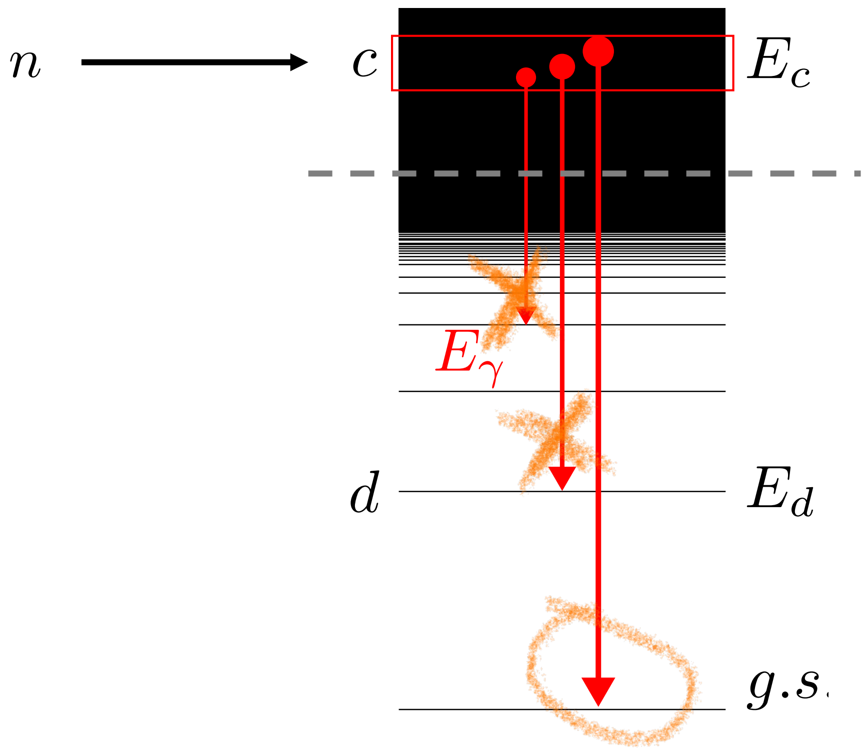


$$T_{c\to d} = \frac{2\pi}{\hbar}\left|\langle\,_d|\hat{H}_{\text{ext.}}|\,_c\rangle\right|^2$$

(Energy-averaging)

$f_d^{XL}(E_\gamma)$ **Radiative strength function**

Brink-Axel hypothesis approximates the general RSF with the ground-state RSF



$$T_{c\to d} = \frac{2\pi}{\hbar}\left|\langle \cdot_d|\hat{H}_{\mathrm{ext.}}|\cdot_c\rangle\right|^2$$

$$f_d^{XL}(E_\gamma)$$ Radiative strength function

$$f_{d=g.s.}^{XL}(E_\gamma)$$ **Brink-Axel hypothesis**

Photo-absorption cross section is directly related to $f^{XL}_{d=g.s.}$

$\langle \sigma_{\mathrm{abs}}(E_\gamma) \rangle = C\, E_\gamma\, f^{XL}_{d=g.s.}(E_\gamma)$

(C handles time reversal)

Radiative decay *to* g.s.:

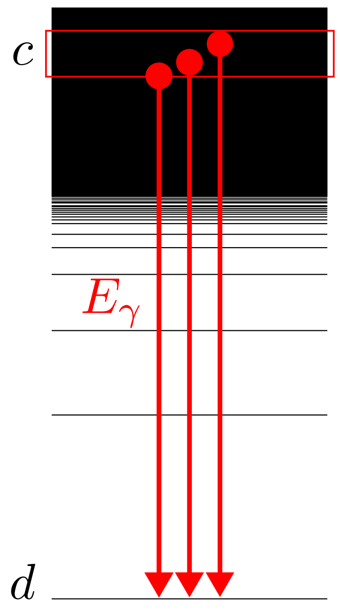
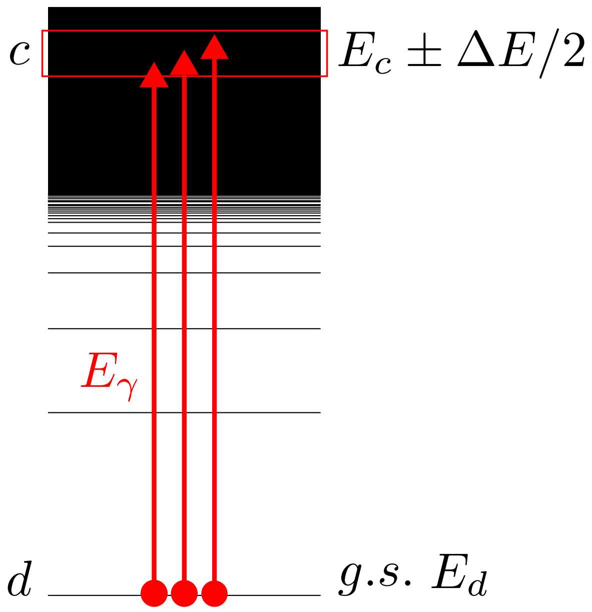
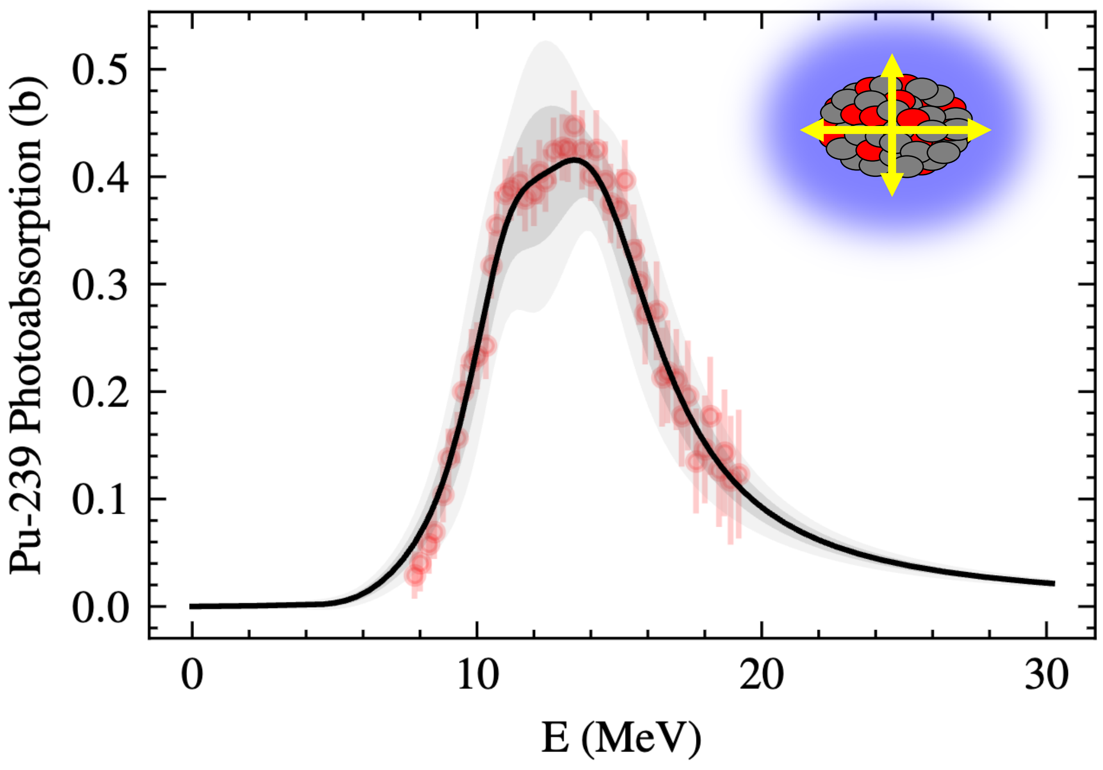


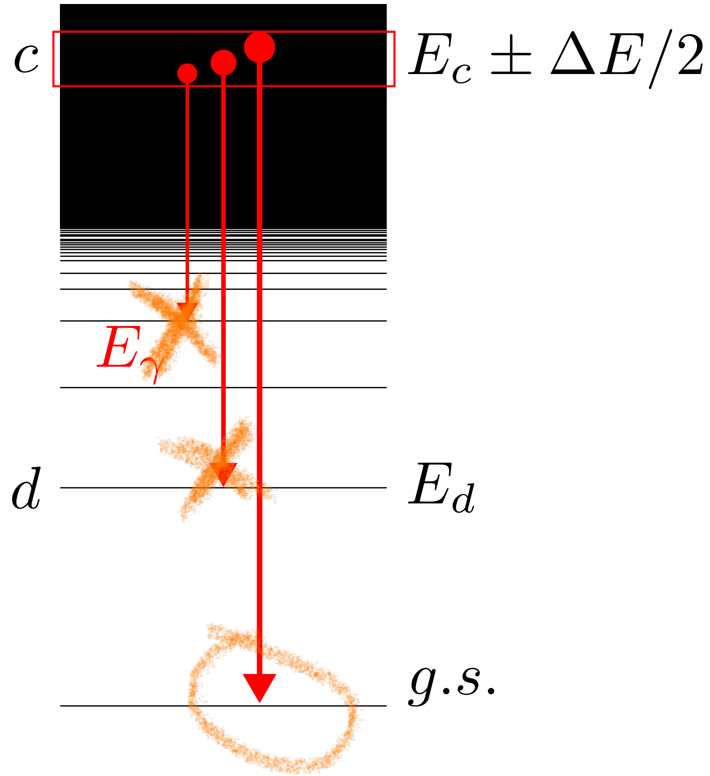
Photo-absorption *from* g.s.:



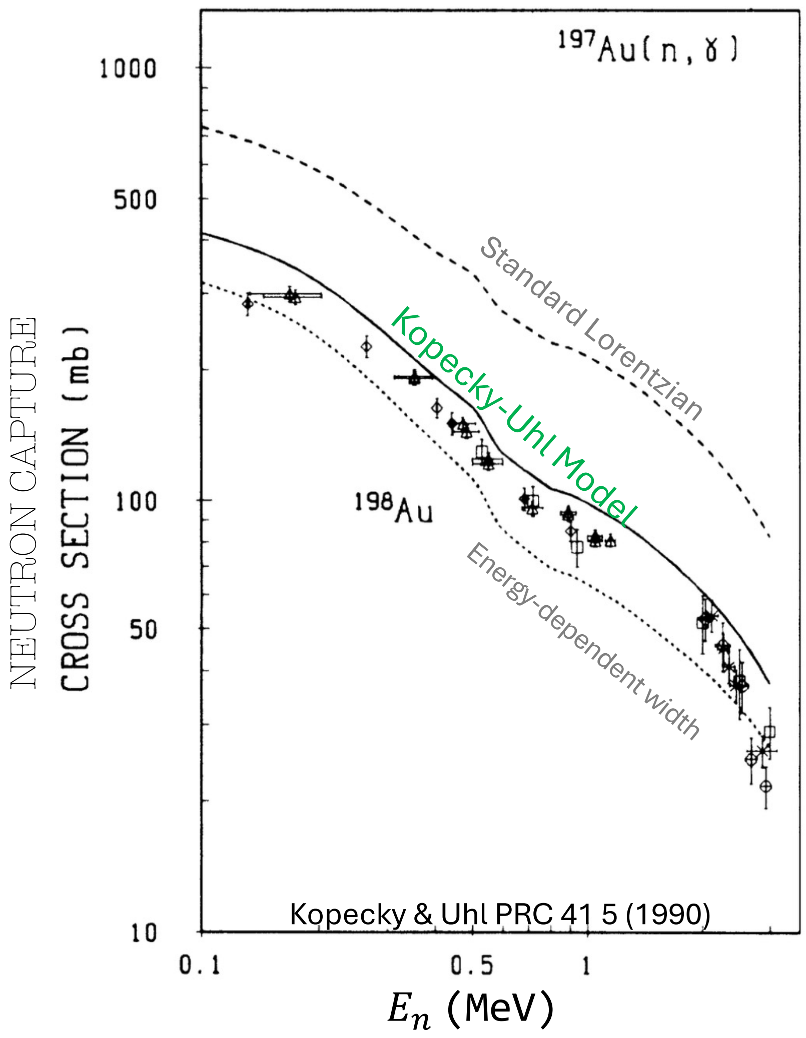
The Giant Dipole Resonance (GDR) is observed in photo-absorption – but it misses something

Lorentzian shape(s) used in Hauser-Feshbach



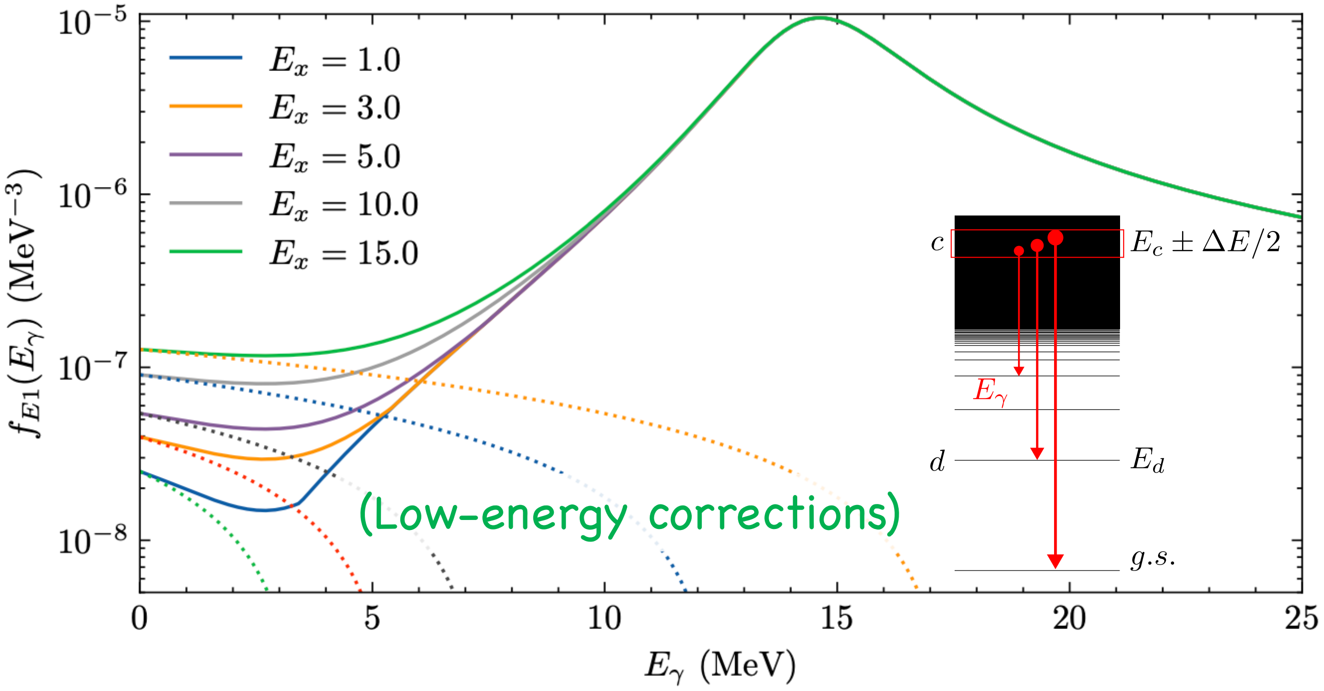


Standard Lorentzian lacks low-energy strength important for reactions



Kopecky-Uhl GSF shape used in Hauser-Feshbach

(Explicit violation of BA hypothesis)



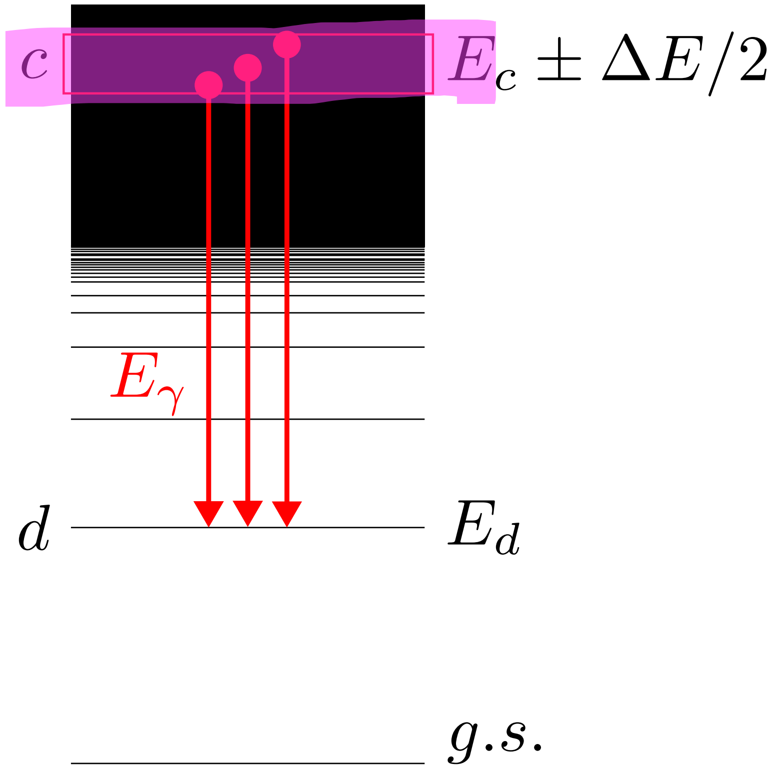
From the Bartholomew definition to a shell-model-friendly definition

$$f^{XL}_{\text{Bartholomew}}(E_\gamma) = \frac{\langle \Gamma^{XL}_{dc} \rangle_{c,j_c}}{E_\gamma^{2L+1}} \rho(E_c, j_c)$$

$$\left(\frac{1}{\cancel{N_c}} \sum_c \Gamma^{XL}_{d \leftarrow c}(E_\gamma) \right) \left(\frac{\cancel{N_c}}{\Delta E} \right)$$

$$f^{XL}_{d,j_c}(E_\gamma) = \frac{1}{E_\gamma^{2L+1}} \frac{1}{\Delta E} \sum_{c'} \delta_{j_{c'} j_c} \Gamma^{XL}_{d \leftarrow c'}$$

Energy-averaged decay width to level “d” as a function of E_γ



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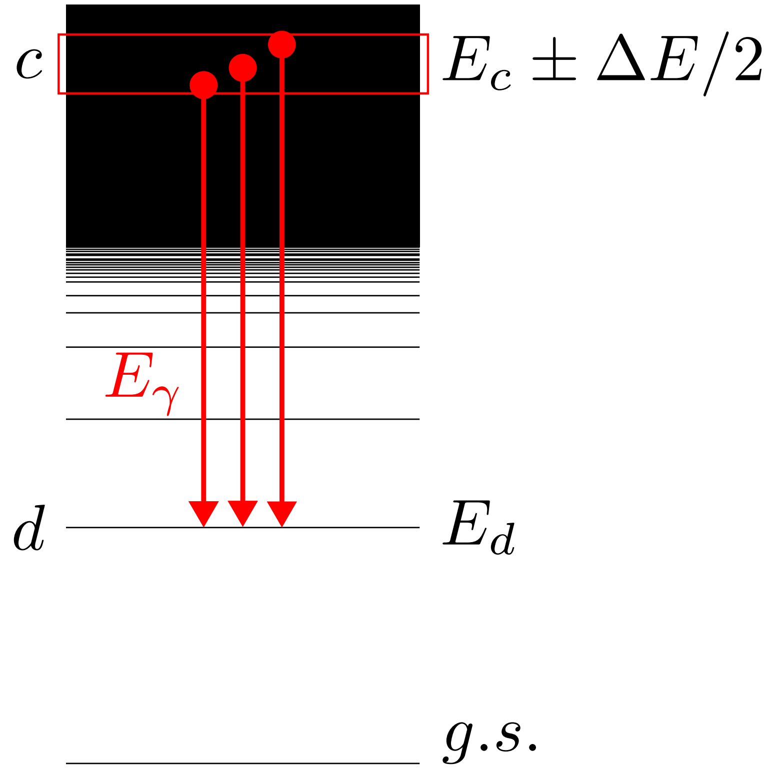
What to do with the dependence on d and j_c?

$$f_{d,j_c}^{XL}(E_\gamma) = \frac{1}{E_\gamma^{2L+1}} \frac{1}{\Delta E} \sum_{c'} \delta_{j_{c'} j_c} \Gamma_{d \leftarrow c'}^{XL}$$



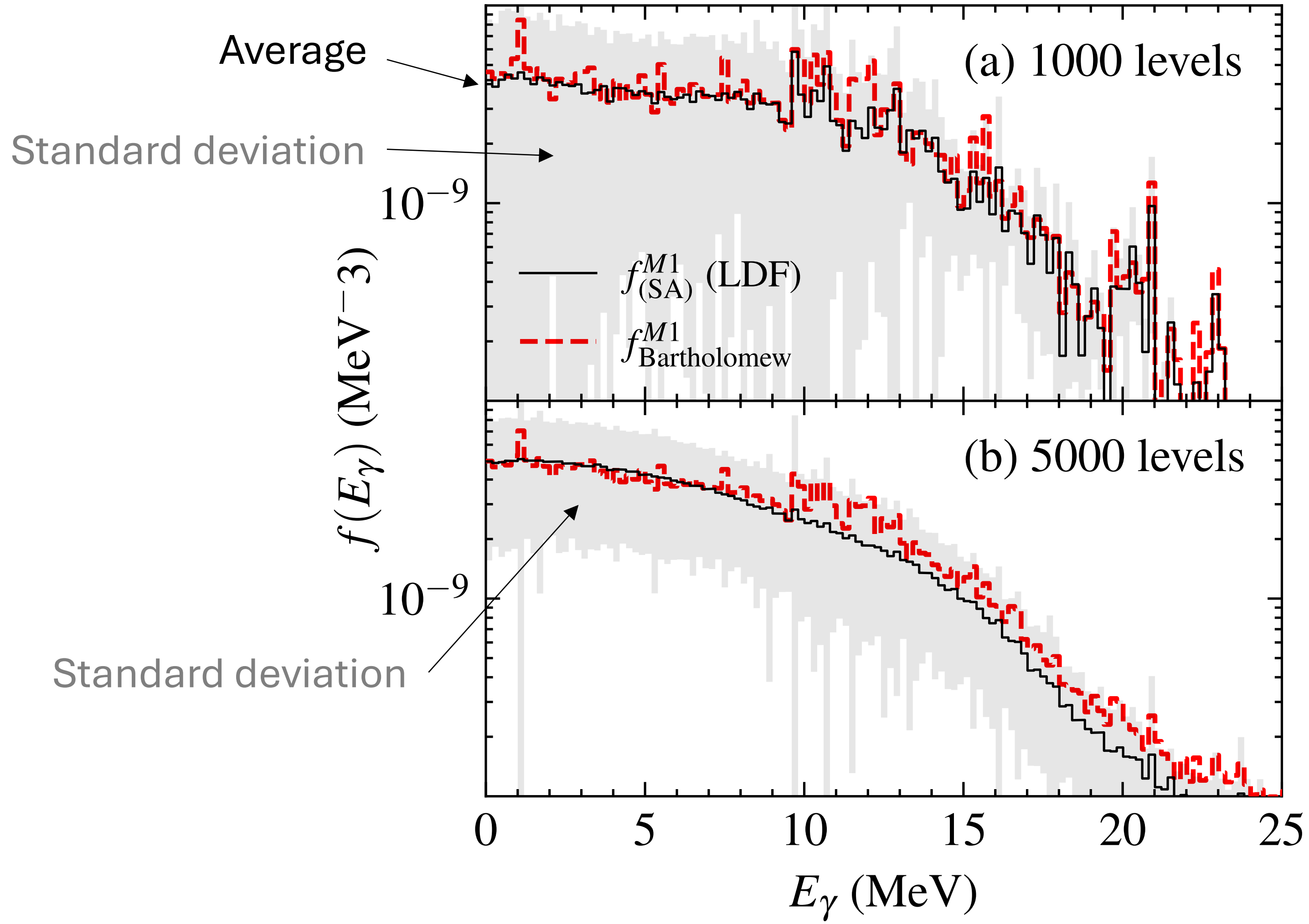
Brink-Axel hypothesis:

- Use any level (e.g. the ground state)
- Average many levels (think Oslo method)

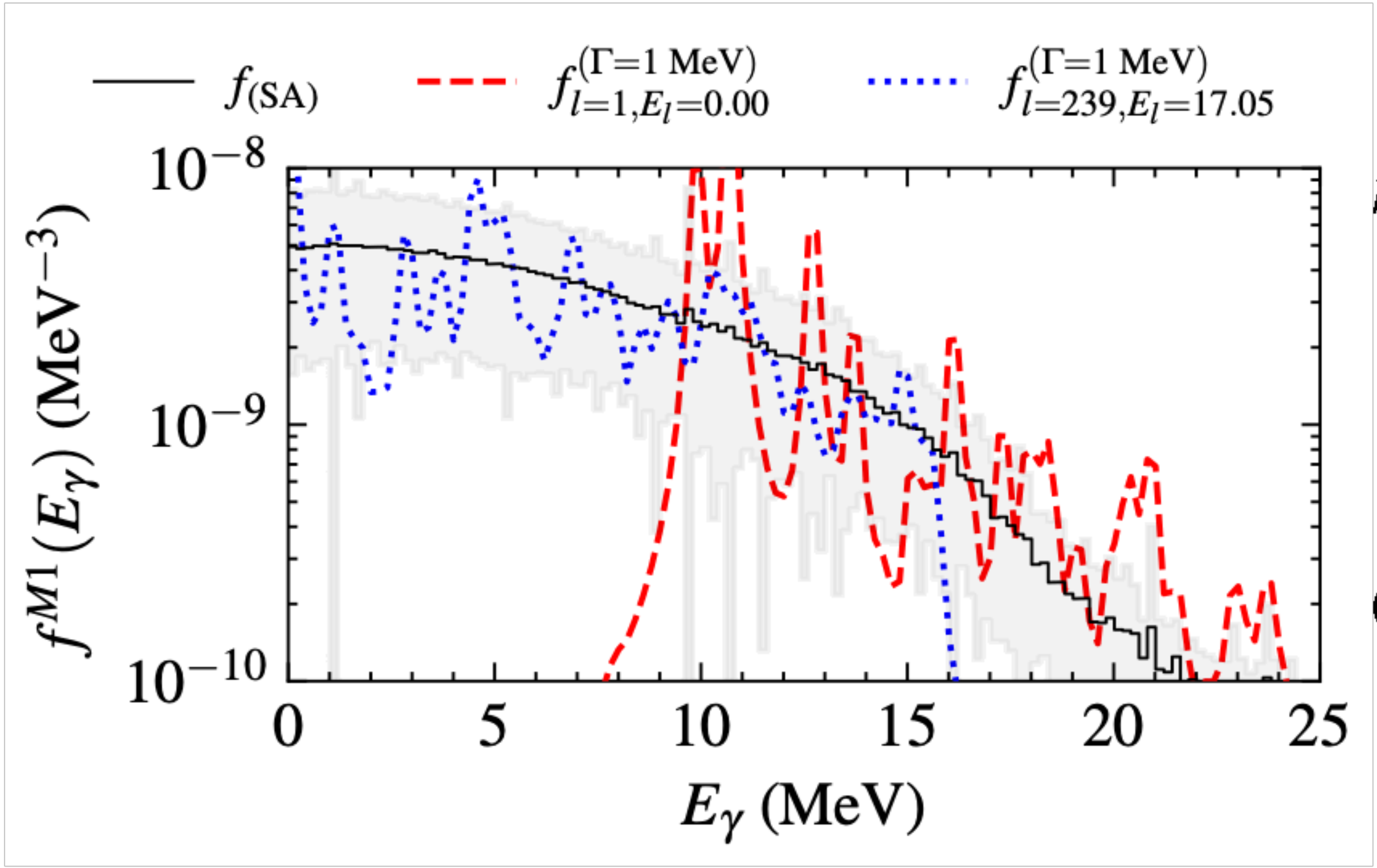
Energy-localized Brink-Axel hypothesis:

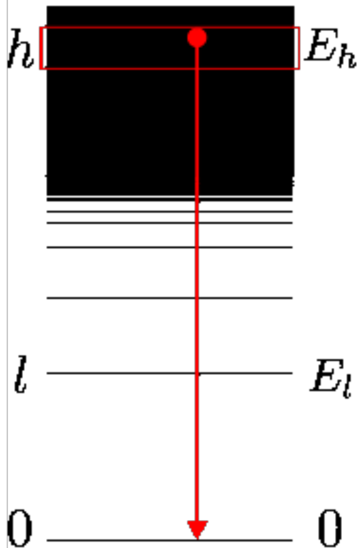
- Average all levels nearby in energy

Brink-Axel Hypothesis: average over levels (d)
^{24}Mg



Simplified definition reveals individual contributions ^{24}Mg





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We can now apply the well-known Lanczos Strength Function (LSF) method

Key advantage:
Instead of computing thousands of wave functions -
(1) Compute *a single wave function*, (2) apply *constant number* of Lanczos iterations
This has already been done for "d=g.s.". The new insight is to apply it to "d > 0".

LSF : computes the strength distribution of an operator by modifying the Lanczos algorithm

Inputs:

$|\psi_d\rangle$: Excited wave function

$|v\rangle = E1|\psi_d\rangle$: Pivot vector

Lanczos iterations:

$\langle\psi_d H\,|v\rangle$
$\langle\psi_d H^2\,|v\rangle$
$\langle\psi_d H^3\,|v\rangle$
…
$\langle\psi_d H^N\, E1\,|\psi_0\rangle$

Outputs:

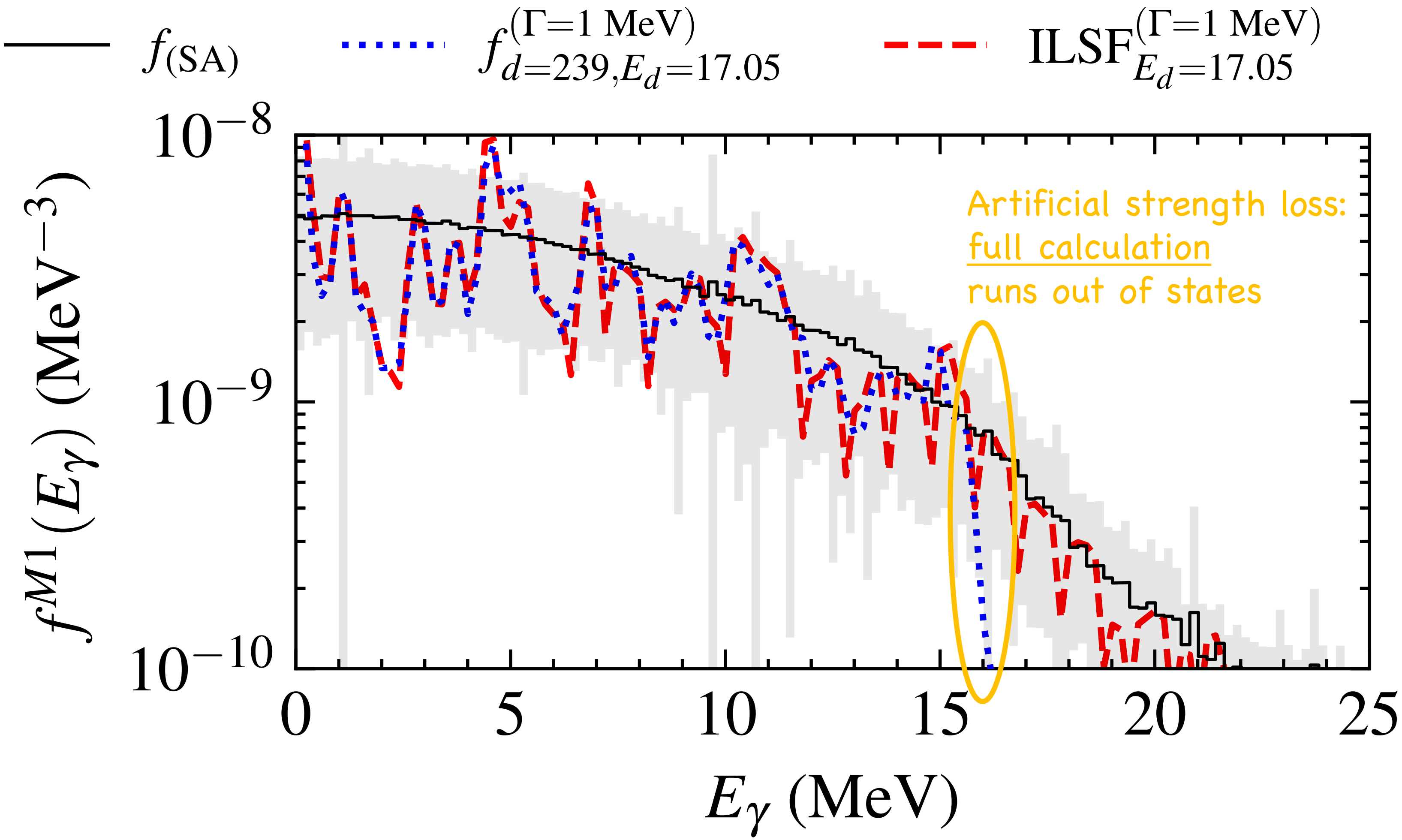
$\langle\psi_{c1}|E1\,|\psi_d\rangle$
$\langle\psi_{c2}|E1\,|\psi_d\rangle$
$\langle\psi_{c3}|E1\,|\psi_d\rangle$
…
$\langle\psi_{cN}|\,E1\,|\psi_d\rangle$

Application:

$$T_{c\to d} = \frac{2\pi}{\hbar}\left|\langle\ _d|\hat{H}_{\text{ext.}}|\ _c\rangle\right|^2$$

$$f^{XL}_{d,j_c}(E_\gamma) = \frac{1}{E_\gamma^{2L+1}}\frac{1}{\Delta E}\sum_{c'}\delta_{j_{c'}j_c}\Gamma^{XL}_{d\leftarrow c'}$$

Apply LSF method to excited states too!
^{24}Mg



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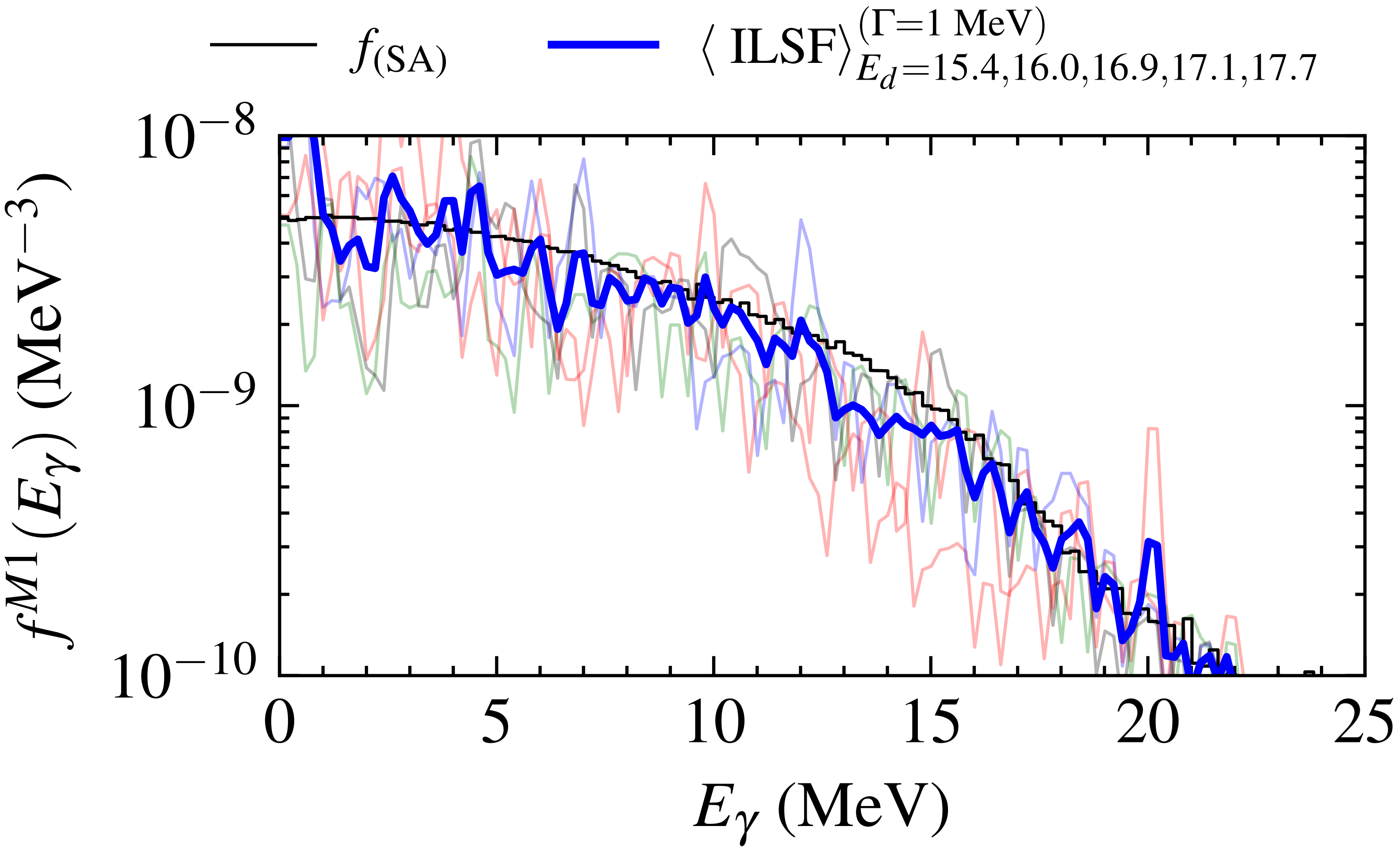
Energy-Localized Brink-Axel Hypothesis (ELBAH) allows us to approximate the RSF with few states



Full calculation:
Converged 5000 wave functions

LSF method:
Converged 5 wave functions
(+200 Lanczos iterations)

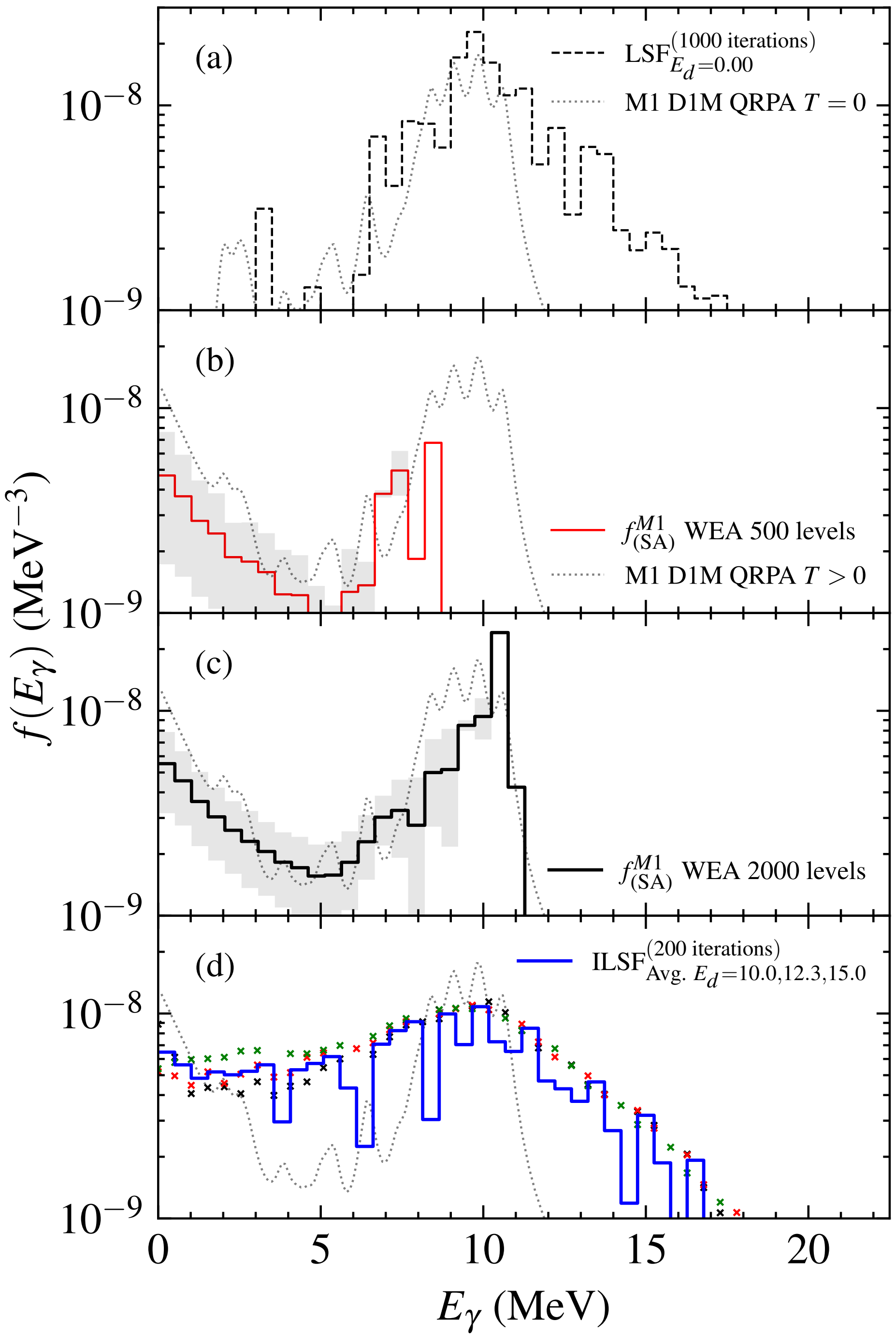
^{56}Fe
M1 shape evoluation?

To ground state

Average over first 500 levels

Average over first 2000 levels

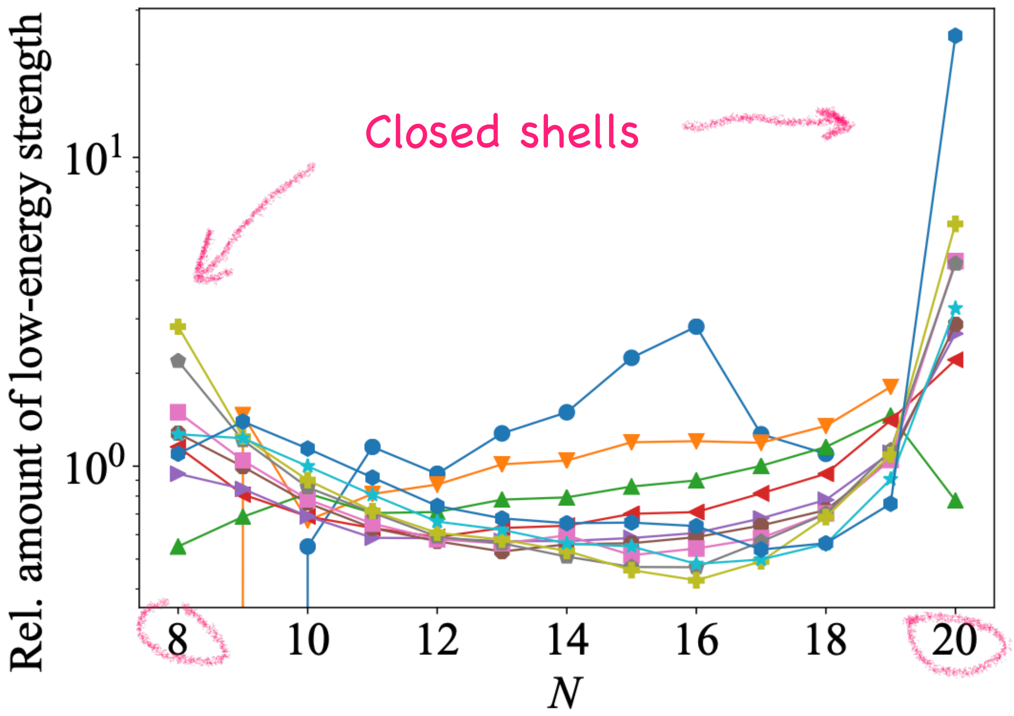
Average over 3 high-energy levels



Evolution of low-energy M1 strength:

(1) nearness to closed shell

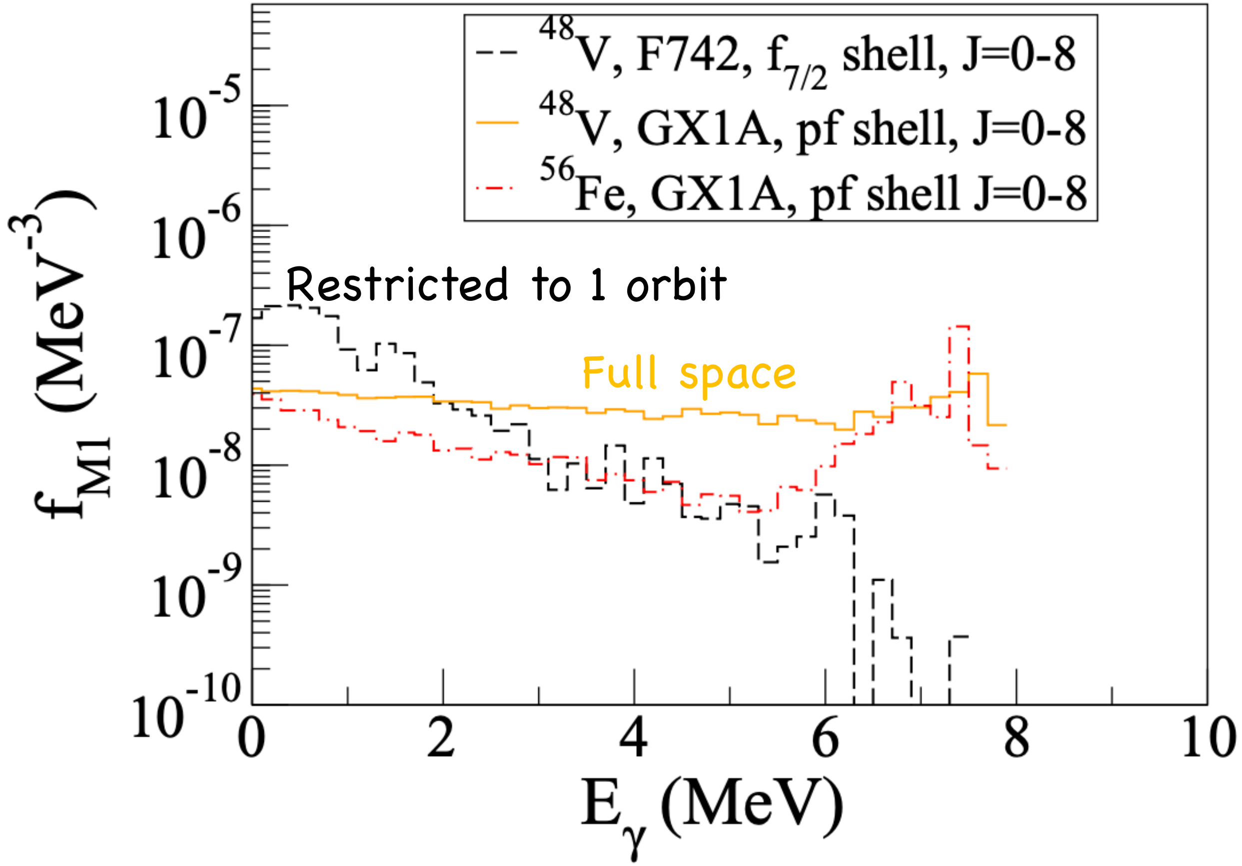
(2) restricted valence space



Midtbø, Larsen, et al. (2018) PRC 98, 064321

sd-shell: biggest M1 upturn near closed shells

Evolution of low-energy M1 strength:
(1) nearness to closed shell
(2) restricted valence space



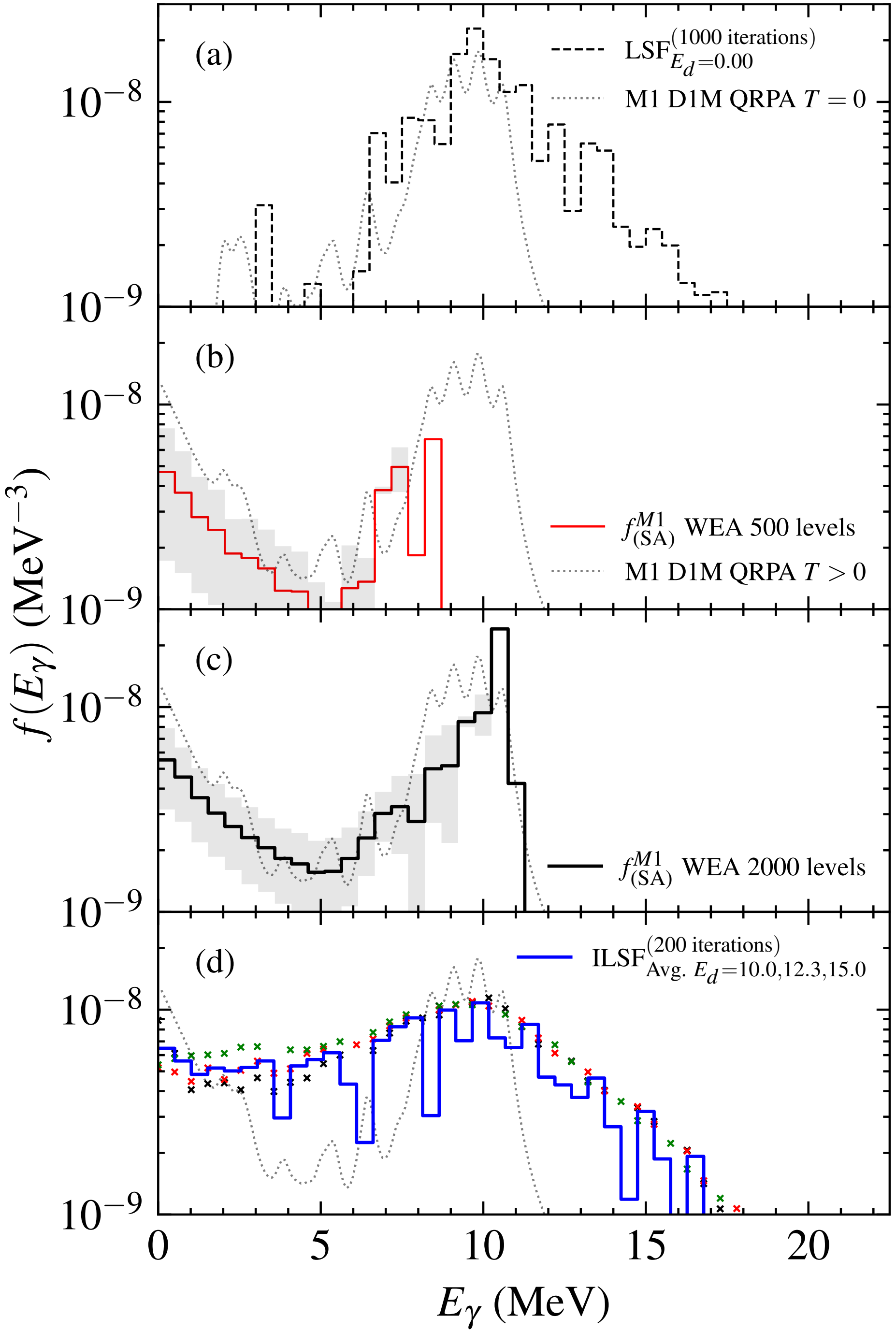
Karampagia, Brown, Zelevinsky (2017)
PRC 95, 024322

pf-shell: exponential shape damped by orbital mixing

M1 shape flattens with energy because single-particle orbits become entangled

^{56}Fe

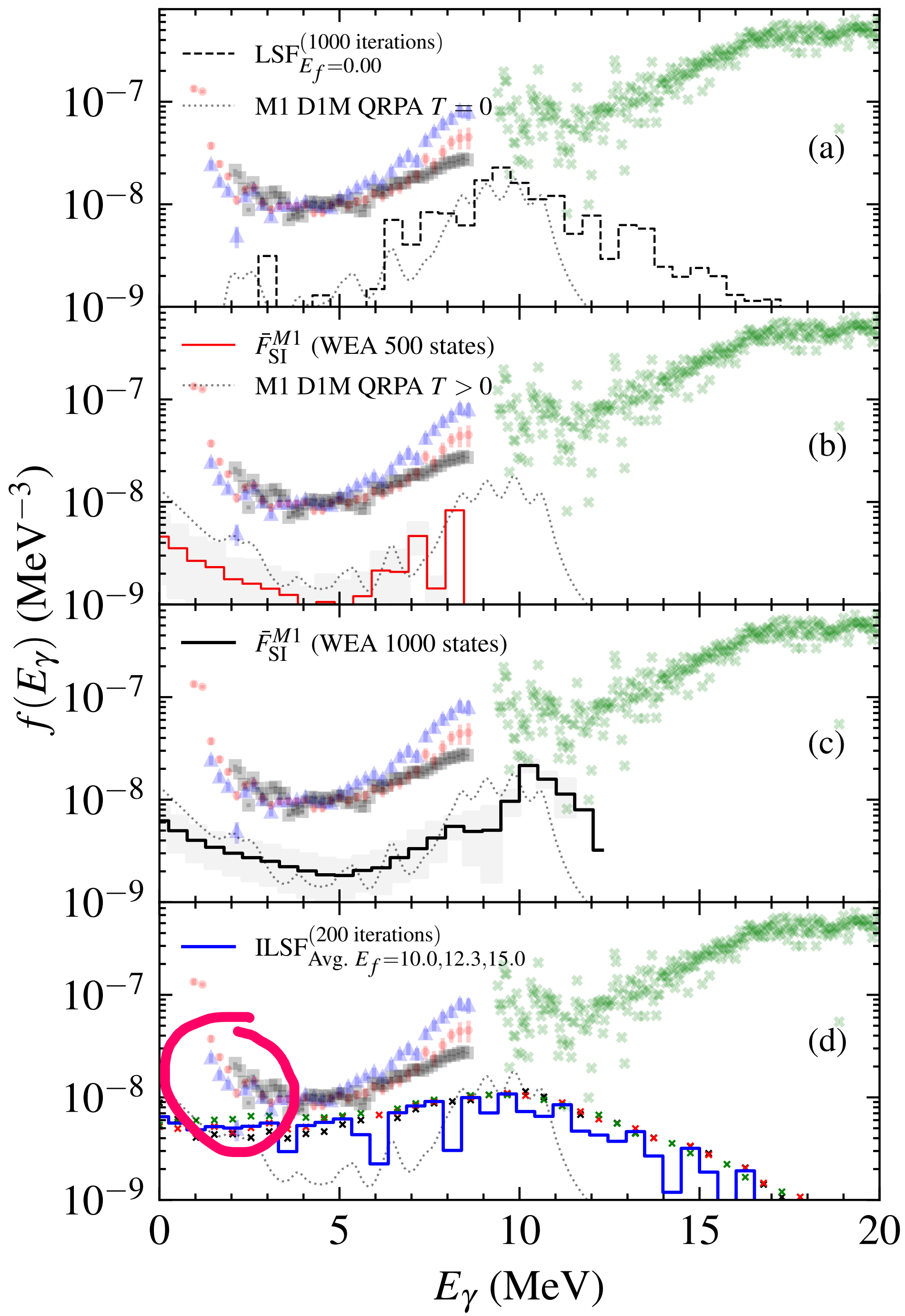
Strength flattens
when decaying to excited states



What about the Oslo-type data?

^{56}Fe

M1 cannot explain observed up-turn according to LSSM



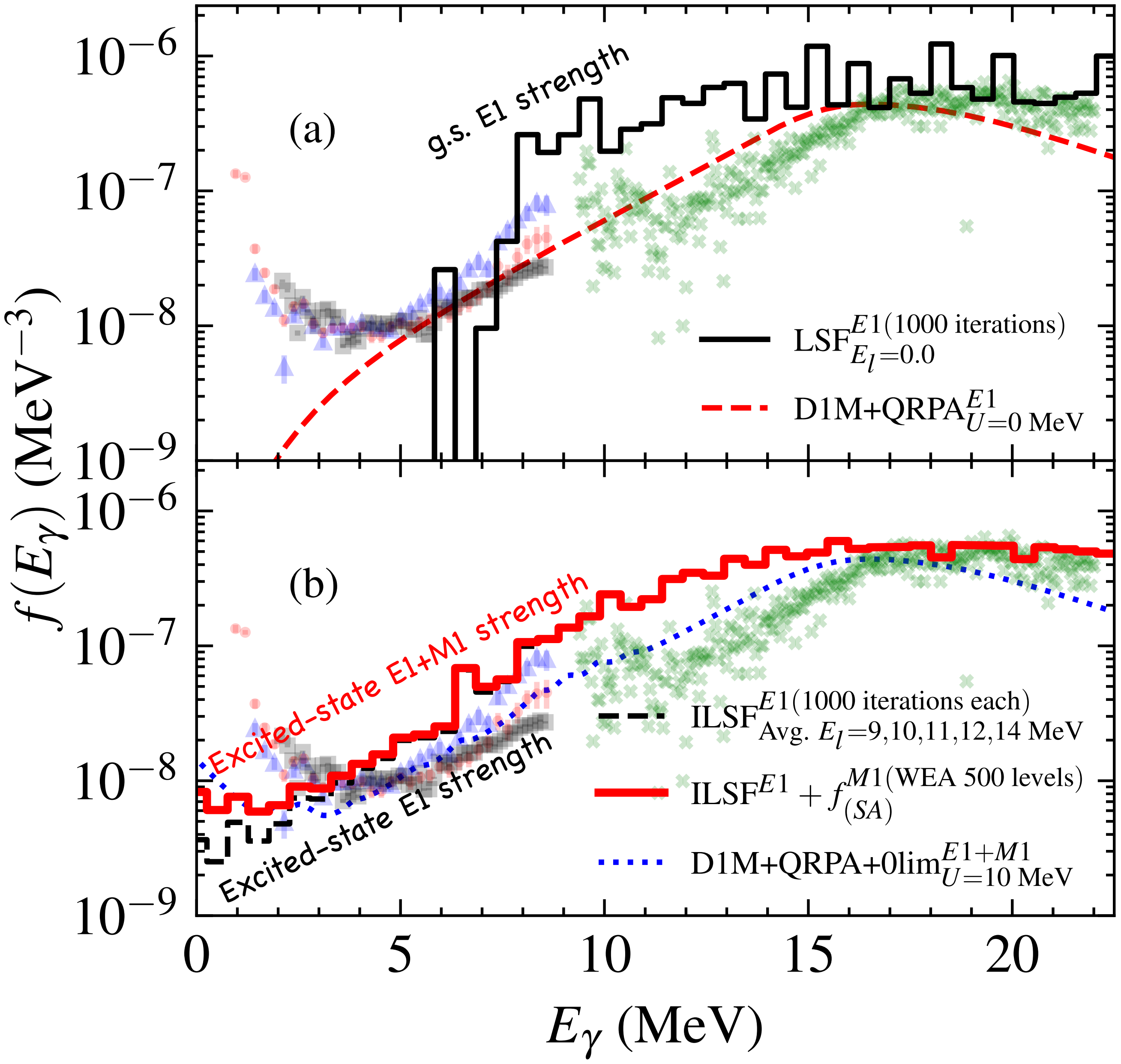
E1 RSF to excited states now possible with LSSM

Fe-56

No adjustment
No folding
sdpf model space (dimension 10^{15})
+ Energy-based configuration truncation (500 M)
+ ILSF method (5 semi-converged wave functions)

For your consideration:

- Interaction (sdpfmu-db) not tuned for pf nuclei
- Missing g9/2 orbit:
    - May shift E1 mean-energy up
    - Should not affect low-energy
- What could generate observed strength?
    - Would need to modify total M1



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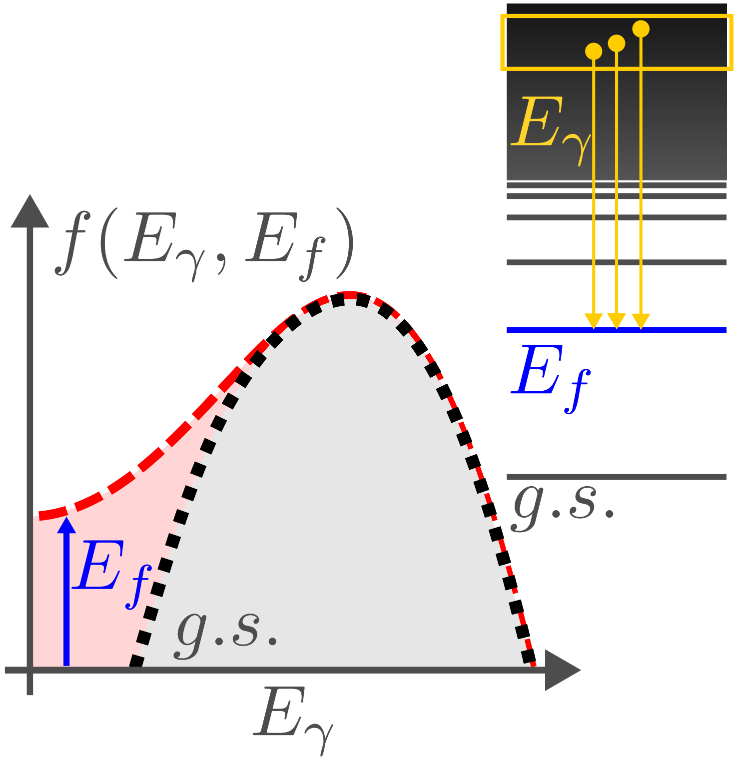
Radiative strength functions and ELBA hypothesis

Takeaways:

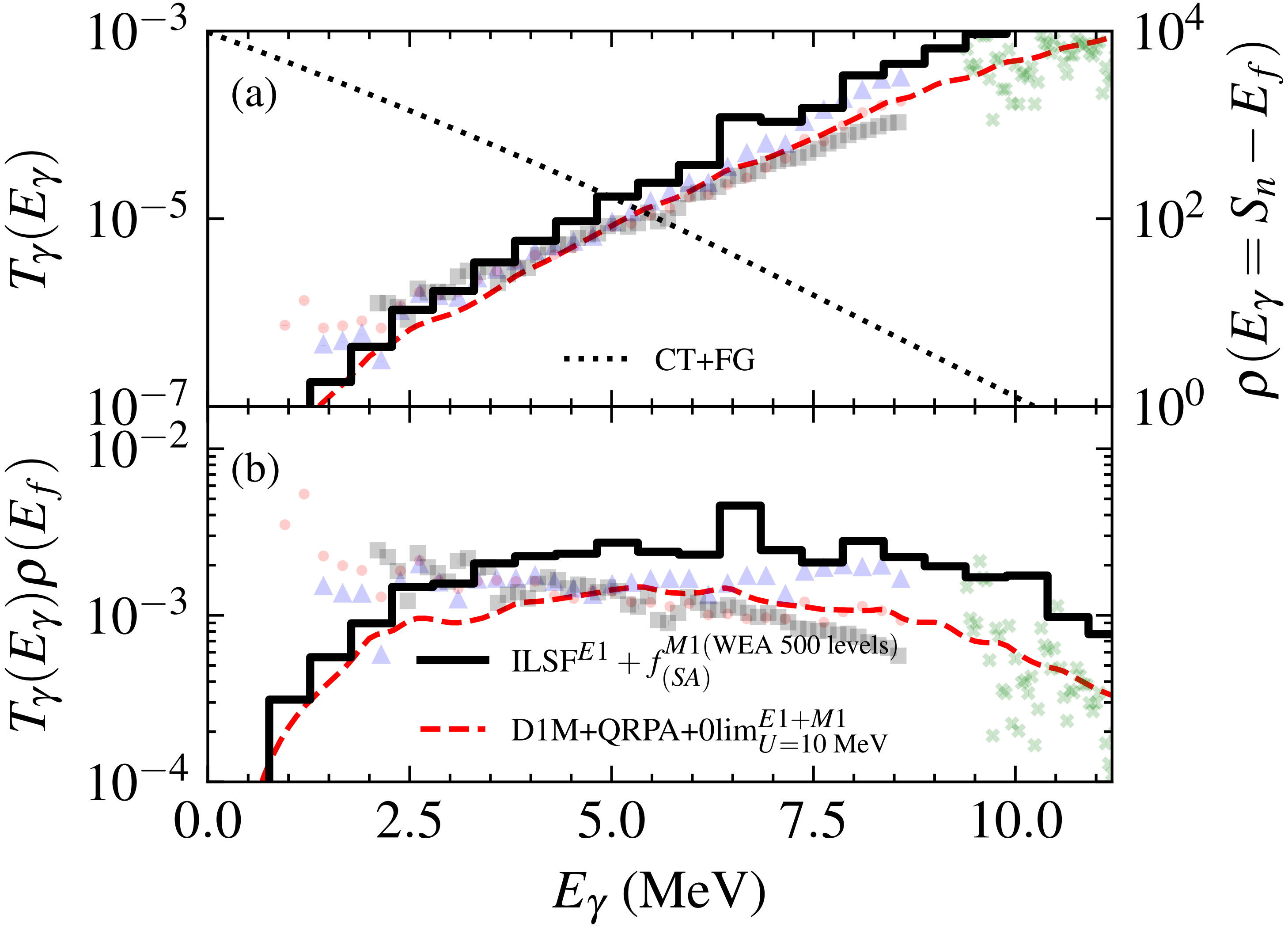
1. Radiative strength (E1 and M1) increases at low E_γ, relative to photo-absorption
   (see Kopecky-Uhl model)

2. Exponential shape (M1) related to single-shell transitions;
   changed by orbital **entanglement**:
   - Open shells (see Midtbø 2018, Karampagia 2017)
   - **Level energy E_f** (this work)

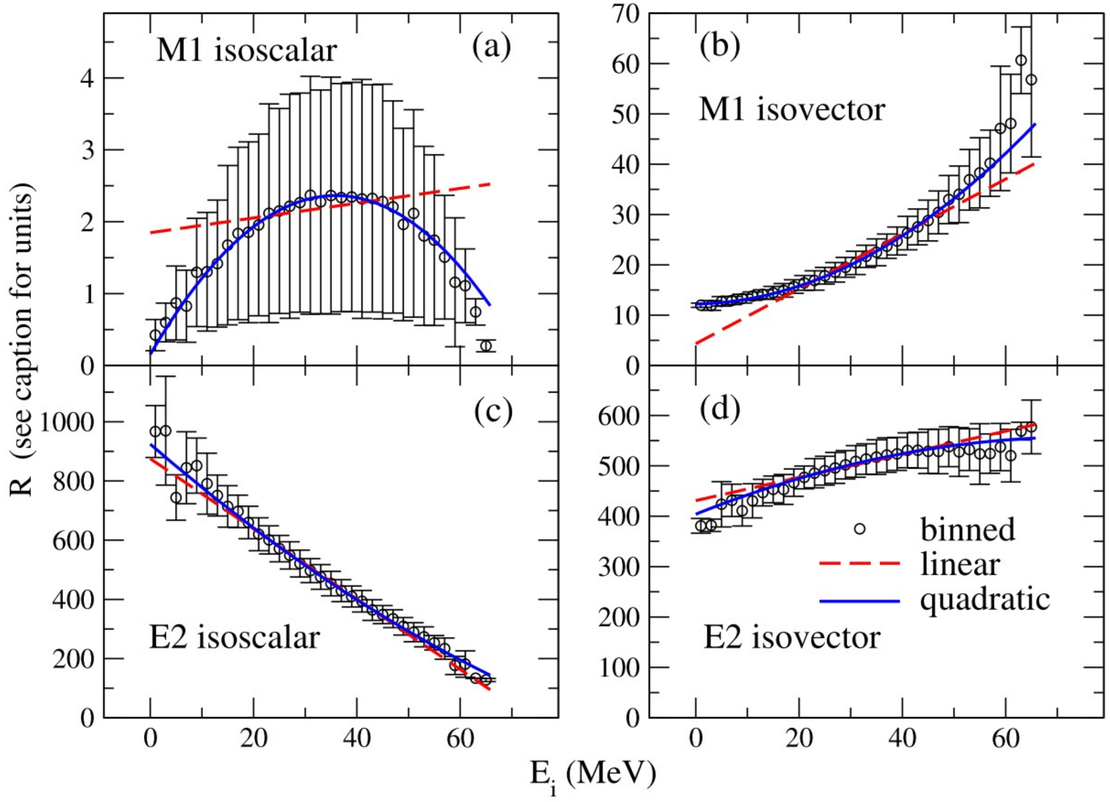
3. The **energy** of the level E_f matters,
   not the number of levels; use **Lanczos**!

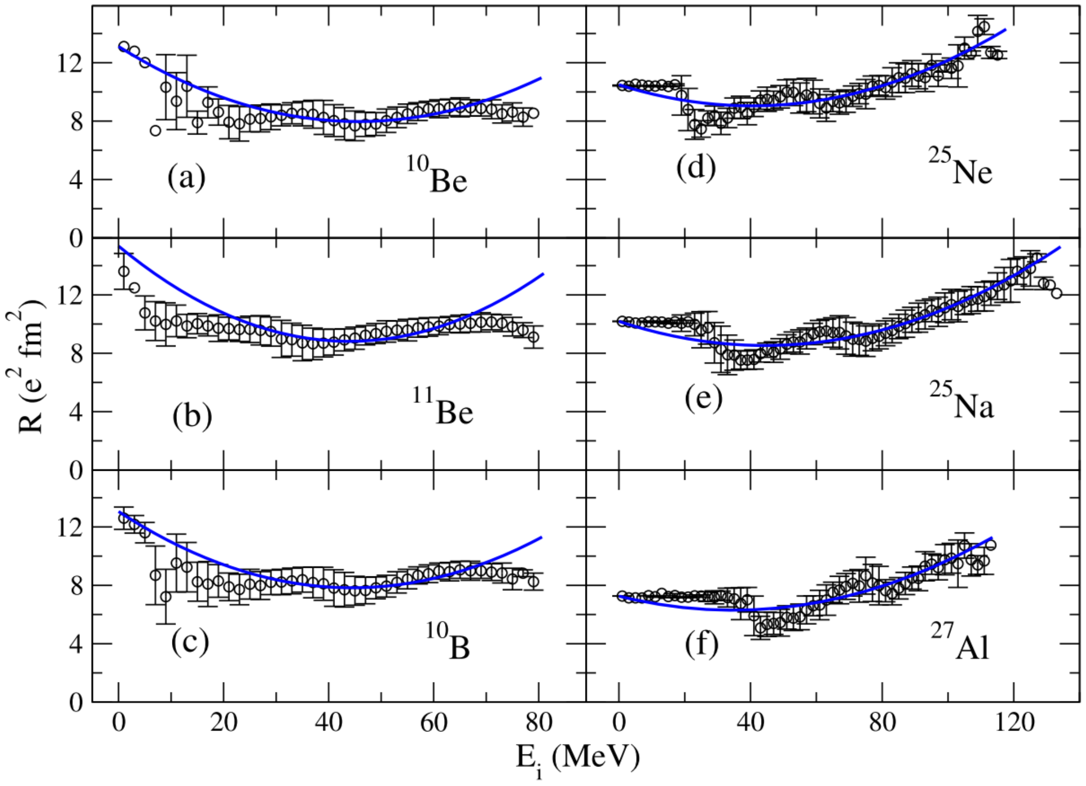


E_γ^3 Makes low-energy differences irrelevant?



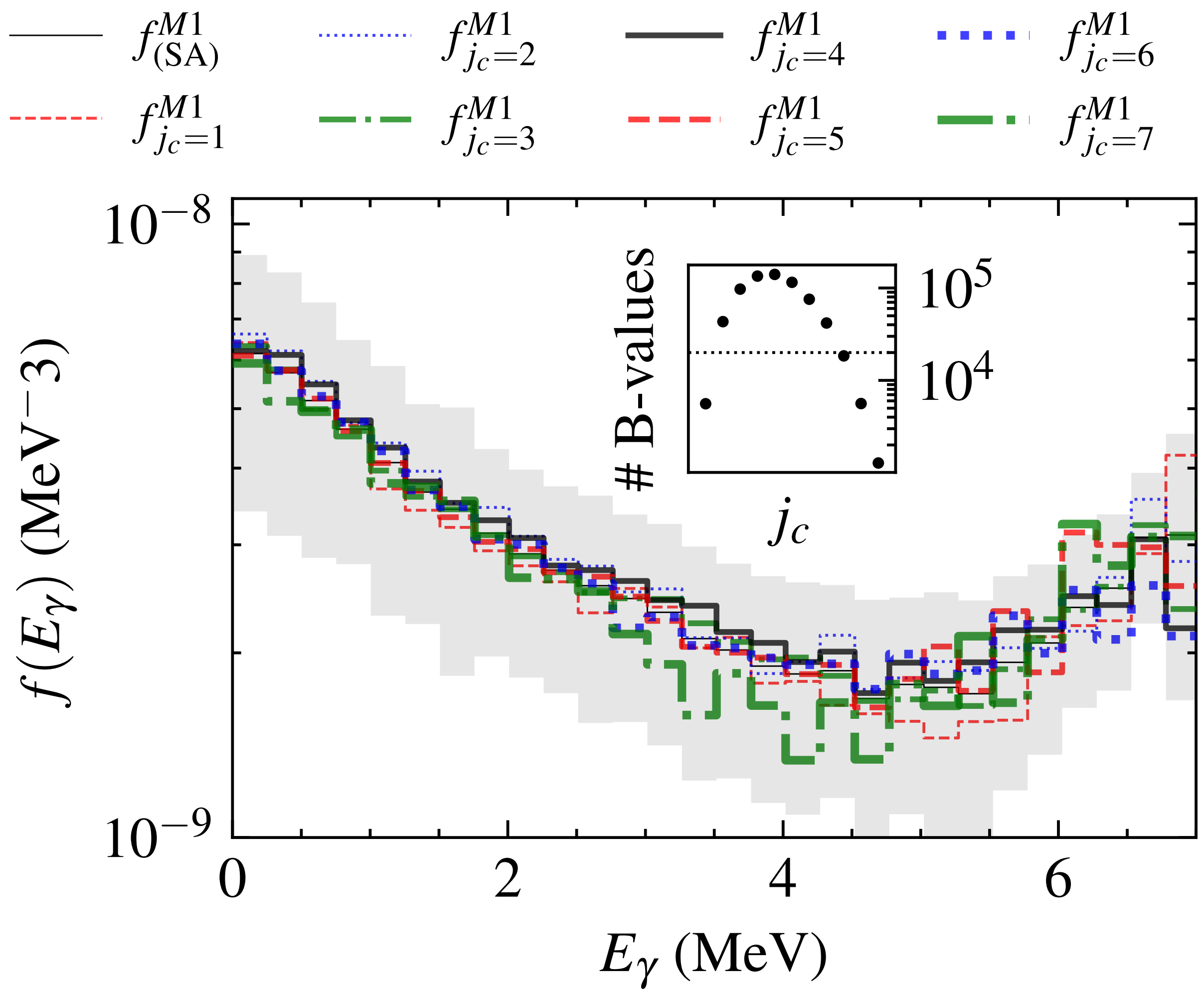
Energy-localized Brink-Axel hypothesis from sum rules





From C. Johnson PLB 750 (2015) 72-75

Spin-independence of RSF seems valid for M1; apparent violations caused by width-fluctuations



$$f^{XL}_{d,j_c}(E_\gamma) = \frac{1}{E_\gamma^{2L+1}} \frac{1}{\Delta E} \sum_{c'} \delta_{j_{c'} j_c} \Gamma^{XL}_{d \leftarrow c'}$$

Averaged over $d < 2000$