Proton and Neutron Approximate Shell Model: Factorization-based Importance Truncation

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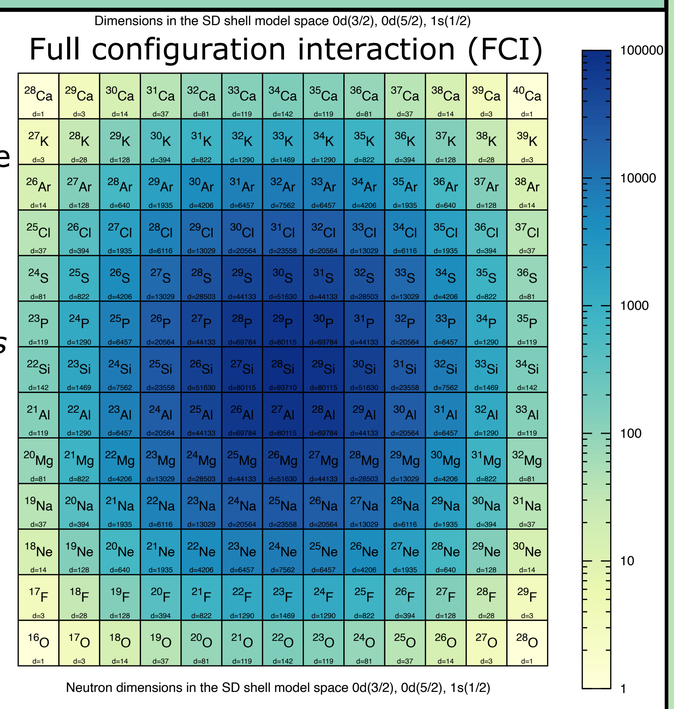
"Do we really need all those numbers?"

The nuclear shell model remain the gold-standard for structure. Its cost reflects this.

Figure 1. Dimension of nuclei in the *sd*-shell model space. (e.g. USDB.) For more complex model spaces, dimensions can exceed 10^{16}

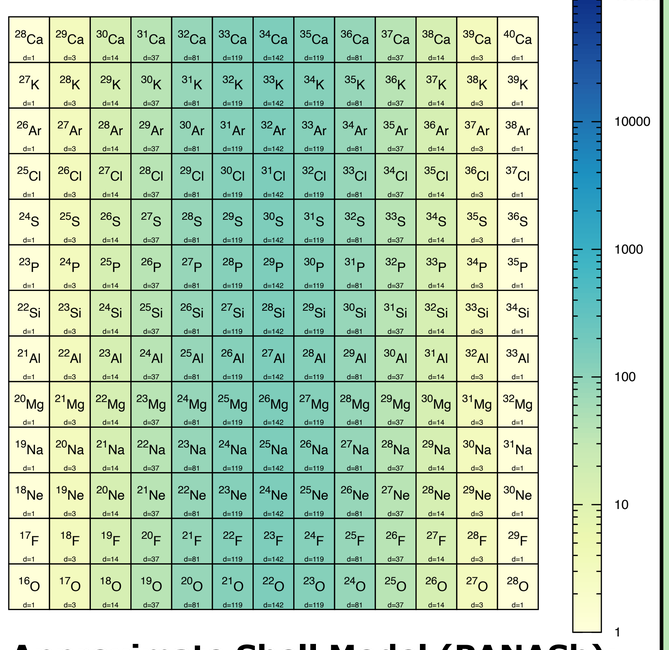
We'd like to make this smaller, so we can actually compute those models.



We were inspired by [PRC.69.024312, Papenbrock, et al. (2004)] and related work to consider a factorization inspired basis...

$$H = H_p + H_n + H_{pp} + H_{nn} + H_{pn}$$

Figure 2. Dimension of neutron subspace for nuclei in the *sd*-shell model space.



$$(H_n + H_{nn})|\phi^{(n)}\rangle = E_n|\phi^{(n)}\rangle$$

$$(H_p + H_{pp})|\phi^{(p)}\rangle = E_p|\phi^{(p)}\rangle$$

$$H = E_n + E_p + H_{pn}$$

Proton and Neutron Approximate Shell Model (PANASh):

Construct a basis: $|\psi_{ab}\rangle = (|\phi^{(p)}; a : E_p, j_p\rangle \otimes |\phi^{(n)}; b : E_n, j_n\rangle)_J$

Truncate: $E_n^2 + E_p^2 < \epsilon_{cut}^2$

Diagonalize H: $H_{a'b'ab} = \langle\psi_{a'b'}|H|\psi_{ab}\rangle$

Like a "poor man's" Density Matrix Renormalization Group (DMRG) [PRL 69, 2863 White (1992)], we use a partial solution of the Hamiltonian to construct a special, truncated basis

Energy Spectra (^{70}Ge): We outperform PHF with the same interaction and capture tricky third 0+

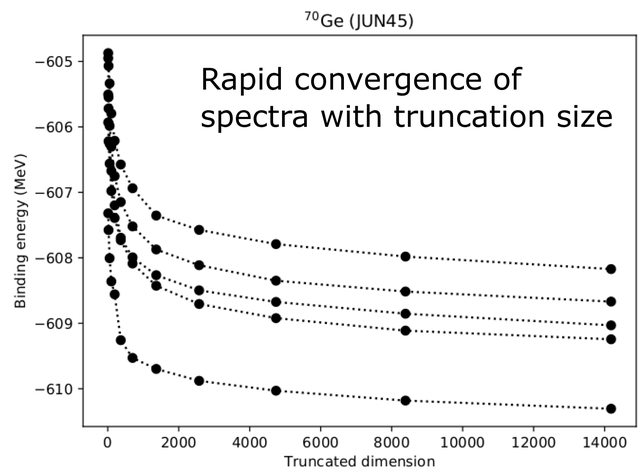


Fig. 1. Convergence of the first few excited states as a function of matrix dimension. M-scheme dimension in this space is 280 million.

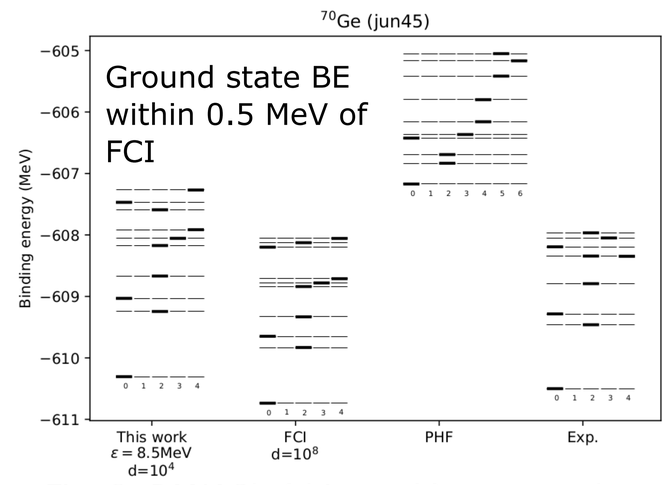


Fig. 2. PANASh (this work) compared to experiment and other methods including projected Hartree Fock [J.Phys.G. 48, 9095107, Lauber, et al. (2021)]. Our method captures complex structure of 70Ge, including third 0+ state.

E2 Transitions (70Ge)

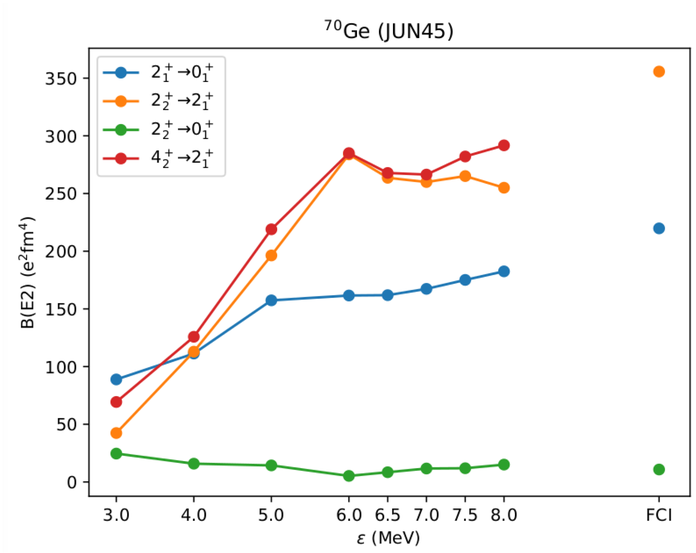


Fig. 3. Convergence of quadrupole transition strengths as a function of the basis cuttoff energy. B(E2)'s are approaching FCI values. We will need to study this further to understand the convergence. Our calculations are bounded by uncertainty of FCI.

Table 1: Quadrupole transition strengths (e^2fm^4)

B(E2)	This work ($\epsilon = 8$ MeV)	FCI	Exp.
$2_0^+ \to 0_1^+$	183	220	358
$2_2^+ \to 2_1^+$	255	356	1950
$4_1^+ \to 2_1^+$	292		410
$2_2^+ \to 0_1^+$	15	11	15

Experimental data from NNDC

What's next: Extrapolation and nuclear reaction observables

Can we extrapolate to the full model-space solutions?

What about other observables relevant to nuclear reaction theory: gamma-ray strength functions (GSFs) and nuclear level densities (NLDs)?

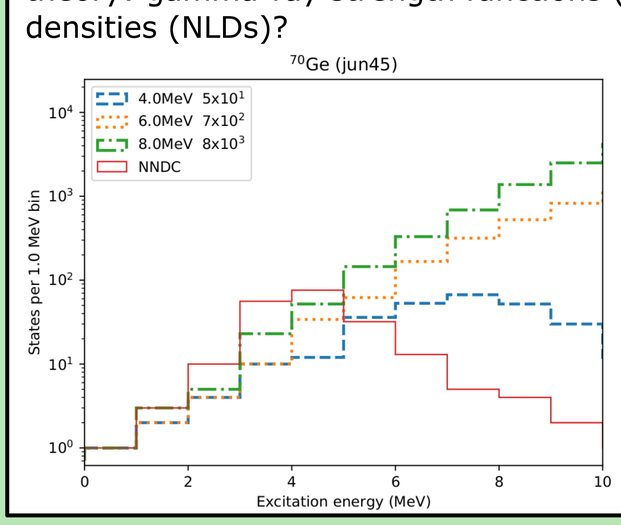


Fig. 5. Preliminary density of levels from PANASh compared to observed levels reported in NNDC.

Gamow Teller Beta Decay (70As to 70Ge)

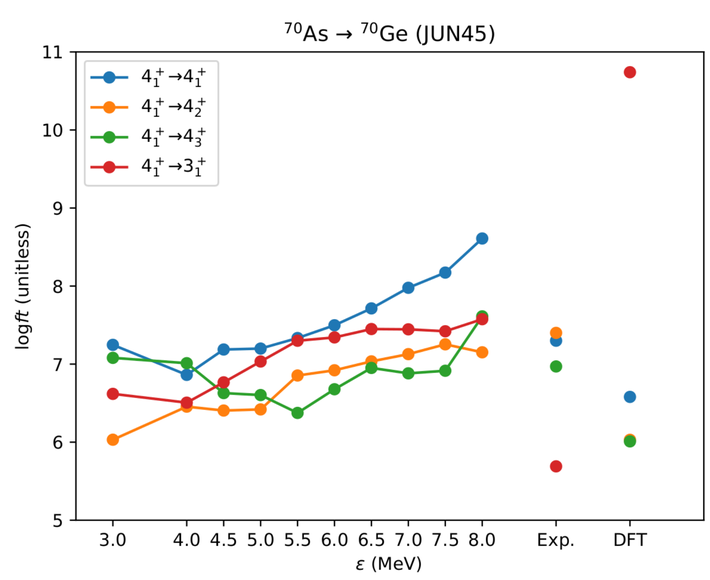


Fig. 4. Convergence of Gamow Teller beta-decay log-ft values compared to experiment and another approximate method: nuclear density functional theory (DFT) and the particle-core coupling scheme from [PRC 105, 044306 (2022)].

Table 2: ^{70}As $\to$ ^{70}Ge β^+ decays. $g_A = 1.27$, $q = 0.6$.

$\log ft$:	This work ($\epsilon = 8$ MeV)	Exp.
$4_1^+ \to 4_1^+$	8.6	7.3
$4_1^+ \to 4_2^+$	7.2	7.4
$4_1^+ \to 4_3^+$	7.6	5.69
$4_1^+ \to 3_1^+$	7.6	6.97

Experimental data from NNDC

We applied our unique proton/neutron factorization scheme to obtain approximate shell model results for the challenging case of ^{70}Ge with multiple observables.

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